Although, the criterion (1), which is minimized to determine $\mathbf{T_y}$ and $\mathbf{T_u}$, is only a nominal stability criterion with respect to the full MIMO system, in the SISO designs of the controllers C_i, robust stability and even robust performance criteria can be specified for the SISO systems $\overline{G}_{dii}$. From a pragmatic point of view, this may be enough to assure in a practical application robust stability or robust performance and this is also implicitly assumed in the paper.

3. Conclusion

The paper of Vaes et al. outlines a procedure to reduce full MIMO controller design to a SVD controller. The main issue is the determination of the constant transformation matrices $\mathbf{T_y}$ and $\mathbf{T_u}$. By optimizing the nominal stability criterion for decentralized controllers, the success rate with respect to nominal stability of the SVD controller is maximized. Despite the fact that from a theoretical point of view no robust stability of the MIMO controller can be guaranteed, the practical example shows that by incorporating robust stability into the design of the SISO controllers, robust stability of the MIMO system is achieved.

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Discussion on: "Comparison of Different Multivariable Control Design Methods Applied on Half Car Test Setup"

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The paper focuses on the replication of a car test drive in a hydraulic test rig. This is basically a tracking problem, where the control signals in the rig should reproduce, as accurately as possible, the acceleration and forces measured in a test drive of the car. The authors propose, and compare, three different multivariable control design approaches to achieve that goal. What makes this article interesting is that it brings together modelling and control design to face the challenge of a real problem. The article is also valuable for its pedagogical pace.

We first comment on some key modelling issues. The authors must deal with the common dilemma of how complex a model should be, since the complexity of the model strongly impacts the control design effort. This dilemma is related to the order of the entries in a matrix transfer function and to the order of the resulting MIMO model. There are two significant statements in the paper regarding this issue. The first one reads, 'Better fits can be obtained with higher order models, but that is not advisable because (i) the estimation of the uncertainty is only approximate, so a more accurate approximation makes no sense (ii) this complicates the controller synthesis and yields unnecessary high order controllers'. Also, the paper reads, 'Reducing the order in the identification process results in less accurate models, and consequently a large uncertainty and very conservative controllers'. However, as shown in Ref. [5] for certain

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model structures, it is best to fit a maximum likelihood (ML) model of the right order and, if required, a low order model can be obtained by model order reduction. Furthermore, as discussed in Ref. [2], if the reduced order model is a smooth function of the full order model (for instance, satisfying an $\mathcal{L}_2$ criterion), then this reduced order model is also a ML estimate, and hence it is also of optimal accuracy. Therefore, an alternative to the approach followed in the paper, might be along these lines, in order to provide a low order model with narrower uncertainty leading, as a consequence, to less conservative controllers.

Another interesting question concerns the black box model approach followed throughout the paper. Since the nonlinearities of this rig are so well pinpointed and available for measurement, a black-box approach might not be the best path to follow. We believe that this rig would provide a good opportunity to attempt a semi-physical model [3]. That is, a gray box modelling approach. A phenomenological model (white-box) can provide an enlightening conceptual organisation of the relevant rig dynamics which, after approximating damper nonlinearities (Fig. 5a of the article) with ad hoc polynomials, could be dealt with the Ritt algorithm of elimination theory [6,7] to get rid of intermediate (not measured) variables while keeping the solution space unaltered. A suitable discretization would produce a parsimonious discrete-time model structure. Depending upon final model structure properties, the parameter estimation may proceed with either some analytical form of least squares, or a numerical technique such as Newton–Raphson. The *a priori* knowledge naturally embedded in the non-linear regressors, yields a model with a broader range of validity than black-box modelling, effectively reducing the uncertainty regions.

Although the modelling problem is fundamental in the control design process, the central subject in the paper is the MIMO control design problem. Three approaches are presented in the paper, one of them, the μ-synthesis method, is basically used as a benchmark to assess the performances delivered by the other two.

The static and the dynamic decoupling strategies are in fact two ideas which are encompassed in the principle of inversion, which underlies every control design strategy.

Consider a stable full MIMO system described by a discrete-time linear transfer function $\mathbf{G}(z)$. Then, the idea of inversion in control design is made explicit when the Youla parametrization of all stabilizing controllers is used. Indeed, in this approach, the Youla parameter $\mathbf{Q}(z)$ must be chosen to be a good inverse of $\mathbf{G}(z)$, at least in the bandwidth of interest [1]. In other words, if the complementary sensitivity, which describes the tracking performance, is denoted by $\mathbf{T}(z)$, then $\mathbf{T}(z) = \mathbf{G}(z)\mathbf{Q}(z)$.

On the other hand, the traditional one-degree-of-freedom feedback control implicitly implements the model inversion on choosing a controller $\mathbf{C}(z)$ which has large gain in that bandwidth (in particular, it usually has infinite gain at zero frequency). It is convenient to keep in mind that the difficulties to achieve a good control almost always originate in the limitations to build, explicitly or implicitly, a good plant inverse. When dealing with MIMO systems, one option is to organize the construction of the inverse in two steps: decoupling followed by a decentralized design. That is, the strategy followed in the paper. Ideally $\mathbf{T}(e^{j\omega})$ must be approximately the identity matrix $\mathbf{I}$ in the bandwidth of interest. However, as the authors rightly point out that this does not yield a stabilizing control if the plant has nonminimum-phase zeros; also realizability problems arise, owing to relative degrees (delays) in the plant. An additional issue which is not explored in the paper, is that certain nonminimum-phase zeros spill to all channels when full decoupling is enforced [1]. This can be avoided if instead of full decoupling we only aim at a triangular structure. This is particularly simple in the 2×2 case. Then a triangular controller can be designed using a sequential synthesis approach.

When one is prepared to abandon the idea of decentralized control synthesis, an alternative strategy to that proposed in the paper provides a wider framework where other solutions are possible. This strategy is based upon the recognition that the invertible part of the plant must be biproper and minimum-phase. The associated factorization of the plant model can be performed using unitary interactors (see [4] and the references therein). Then a wide range of controller synthesis methods based explicitly on inversion (via model matching) can be used. In this framework, design parameters are also available to iteratively achieve a robust controller.

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Discussion on: "Comparison of Different Multivariable Control Design Methods Applied on Half Car Test Setup"

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The authors in the paper present an interesting tracking control problem applied on half car test setup. The plant described in Section 2 represents the rear or front car suspension system excited by two hydraulic actuators: the inputs are the references of the hydraulic actuators ($\mathbf{u}_{\mathbf{ff}}(t)$), while the outputs are the measured accelerations of the 'body mass' ($\mathbf{y}(t)$). This is a typical plant for testing new suspension during the design process, or for evaluating performance of semi-active suspension strategies, and it is based in the so called half-car model [2]. The aim is to compute the input signals of the actuators $\mathbf{u}_{\mathbf{ff}}(t)$ in order to reproduce the vertical acceleration of a car chassis $\mathbf{r}(t)$, measured during a test drive. This issue can be formulated as a multi-variable tracking problem where the error measure $\mathbf{J} = \int(\mathbf{r}(t) - \mathbf{y}(t))^2 \mathrm{d}t$ is to be minimized. For actual industry practice the problem with a feedforward iterative procedure, described in the Introduction, the authors propose to add a feedback controller able to outperform the industrial procedure with a drastic reduction in term of number of iteration and, thus, costs.

The control issue is not trivial for two reasons. The former is the presence of strong nonlinearities in the plant, for instance, owing to the magnetorheological damper by Lord comprised by the suspension, whose nonlinear behaviour is deeply discussed in Ref. [4]. The latter is that the plant is a multiple-input-multiple-output (MIMO) system which requires control techniques more sophisticated than for single-input-single-output systems. The main idea of the authors is to consider the plant as a linear MIMO system modelled in the frequency domain by transfer function matrix $\mathbf{G}(j\omega)$, related to the real plant $\mathbf{P}$ as follows:

$$\mathbf{P} = (\mathbf{I} + \mathbf{W_0}\Delta_\mathbf{0})\mathbf{G}$$

The matrix $\mathbf{W_0}\Delta_\mathbf{0}$ is a measure of the uncertainties of the model owing to the nonlinearities of the system. The robust control paradigm based on H_∞ norm is able to ensure rapid convergence of the feedforward–feedback procedure although the nonlinearities.

In Section 3 the design issue is described. First an identification of matrix $\mathbf{G}(j\omega)$ is performed. Then three different regulators based on robust control techniques are proposed. The first is a MIMO control design based on μ-synthesis. It is outlined how this kind of design guarantees high performance but it is a difficult task in practice owing to MIMO identification and high order of the controller implemented. In order to avoid the MIMO design, the other two techniques proposed are based on static and dynamic decoupling. This yields to consider the plant as two uncoupled SISO systems, so that SISO robust control can be designed.

Finally, the performances of each controller are compared and reported on the basis of the evaluation of the error-to-signal-ratio of the tracking error, made both in the time (**esr**) and frequency domain (**ESR**) :

$$\mathbf{esr} = \frac{\|\mathbf{r} - \mathbf{y}\|_2}{\|\mathbf{r}\|_2} \qquad \mathbf{ESR}(j\omega) = \mathbf{E}(j\omega)\mathbf{R}^{-1}(j\omega)$$

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