“Averaging Level Control of Multiple Tanks: A Passivity Based Approach”

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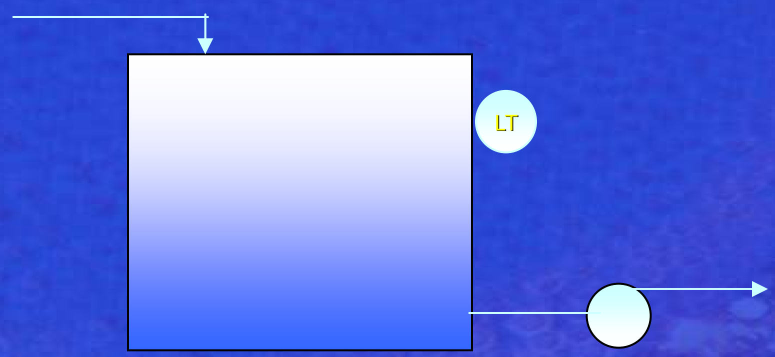
UTFSM-2008

Outline

- The averaging level control problem
- Controller design using IDA-PBC
- Single tank
- Cascade of tanks
- Some simulation results
- Final remarks

The averaging level control problem



$$\dot{V} = Q_F - Q_o$$

$$V_{\min} \leq V \leq V_{\max}$$

The problem:

Find a smooth outlet flow so that the inequalities associated to the level are satisfied. I this way, the downstream effect of the inlet flow disturbances is minimized.

The averaging level control problem

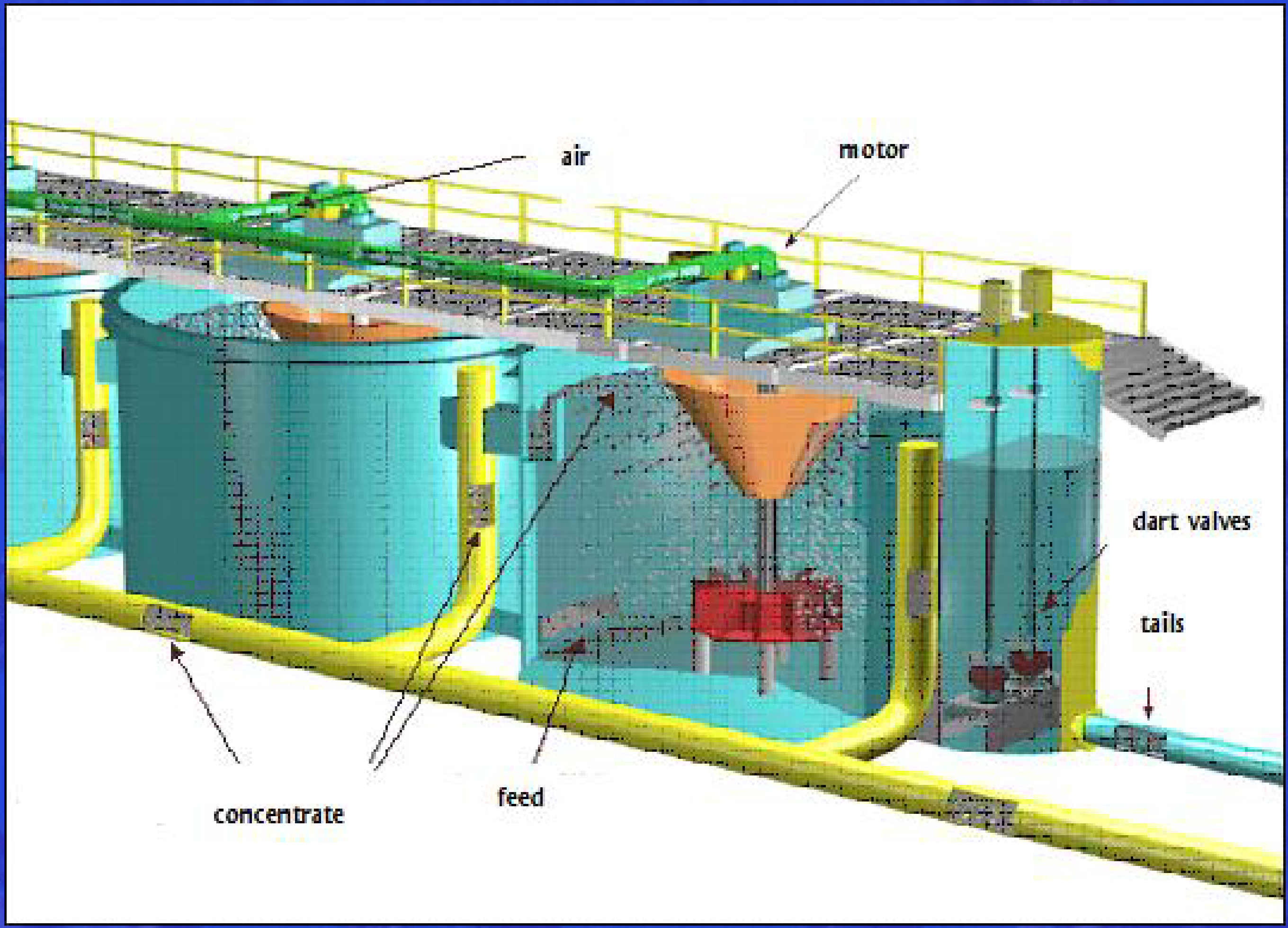
Traditional approaches:

- Controllers with proportional and integral modes or simple intuitively based nonlinear controllers .

- Model predictive controller, where the objective is quantified by the maximum rate of change of outlet fow for a given inlet flow disturbance subject to level constraints

These works consider only one tank processes, and the closed-loop stability analysis is not considered.

The averaging level control problem



Controller design using IDA-PBC

The open-loop system

$$\dot{x} = (J(x,u) - R(x,u))\nabla H + g(x)d$$

$$H(x)$$

The desired closed-loop

$$\dot{x} = (J(x,u) - R(x,u))\nabla H_d$$

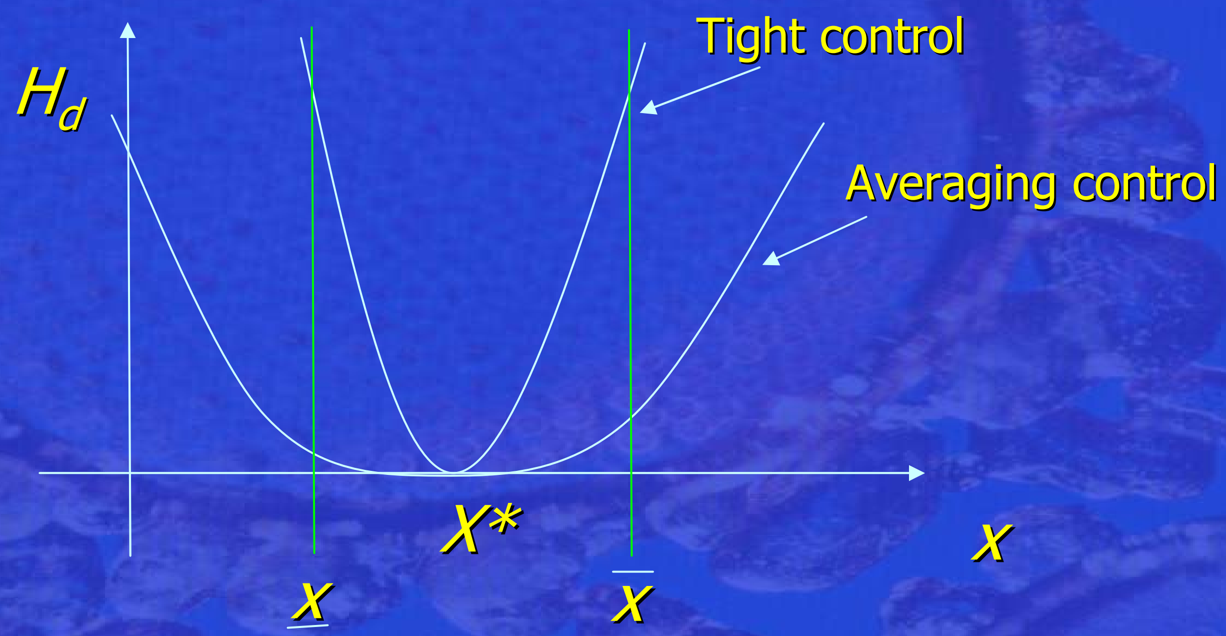
$$H_d(x) = H(x) + H_a(x)$$

Key matching equation

$$(J(x,u) - R(x,u))\nabla H_a = -g(x)d$$

$$\dot{H}_d = -\left(H_d\right)^T R(x,u) H_d \leq 0$$

Controller design using IDA-PBC



The idea:

To shape the total mass functions associated to each tank. For smooth outlet flow a flat energy function around a nominal level, will be required. However, to keep a tight control around the set point, a energy penalizing big errors should be designed.

Controller design using IDA-PBC

$$(J(x,u)-R(x,u))\nabla H_a=-g(x)\hat{d}$$

But $\hat{d}$ is an estimate of an unknown d

$$\dot{x}=(J(x,u)-R(x,u))\nabla H_d+g\tilde{d}$$

$$\tilde{d}=d-\hat{d}$$

Controller design using IDA-PBC

Extended Lyapunov function candidate

$$W(x,\tilde{d}) = H_d(x) + \frac{1}{2\gamma}\ln\left[\frac{(\hat{d}-\overline{d})^{\tau}}{(\hat{d}-\underline{d})^{\tau+1}}\right]$$

$$\dot{\tilde{d}} = \gamma(\hat{d}-\overline{d})(\hat{d}-\underline{d})\left(\nabla H_d(x)\right)^T g$$

$$\dot{W} = -\left(\nabla H_d(x)\right)^T R(x,u)\nabla H_d(x) \le 0$$

Single tank

$$\frac{dV}{dt} = Q_F - Q_o$$

$$V = Ah$$

$$Q_o = f(u)\sqrt{kh + \Delta}$$

$$H = V \geq 0$$

$$\frac{dV}{dt} = Q_F - Q_o \nabla H$$

$$H_d(x) = H(x) + H_a(x) = V + \phi(V, V^*, V_{min}, V_{max})$$

$$f(u) = -\frac{Q_F}{\frac{\partial \phi}{\partial V}\sqrt{kh + \Delta}}$$

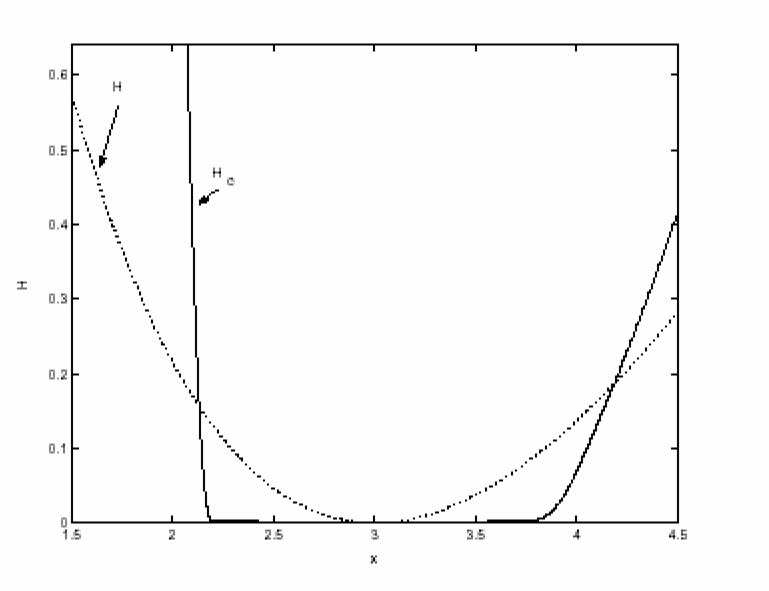
Single tank

$$\phi(V, V^*) = -V^* \ln(V)$$

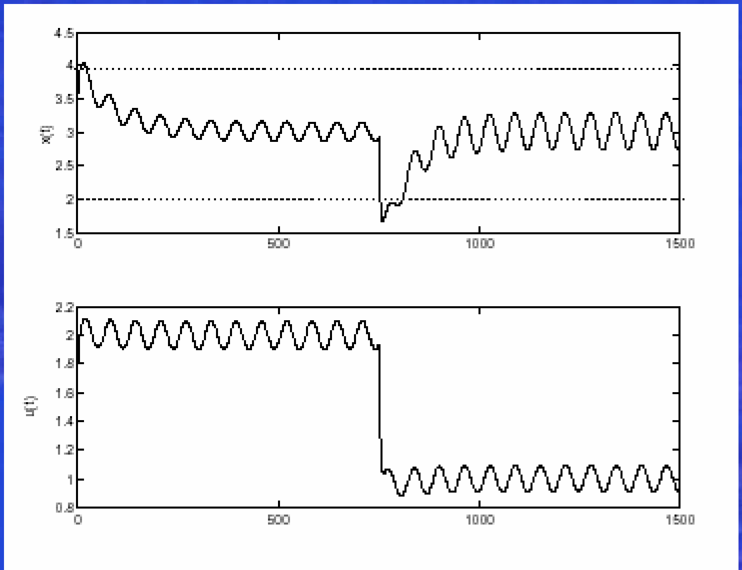
$$f(u) = \frac{Q_F}{V^* \sqrt{kh + \Delta}} V$$

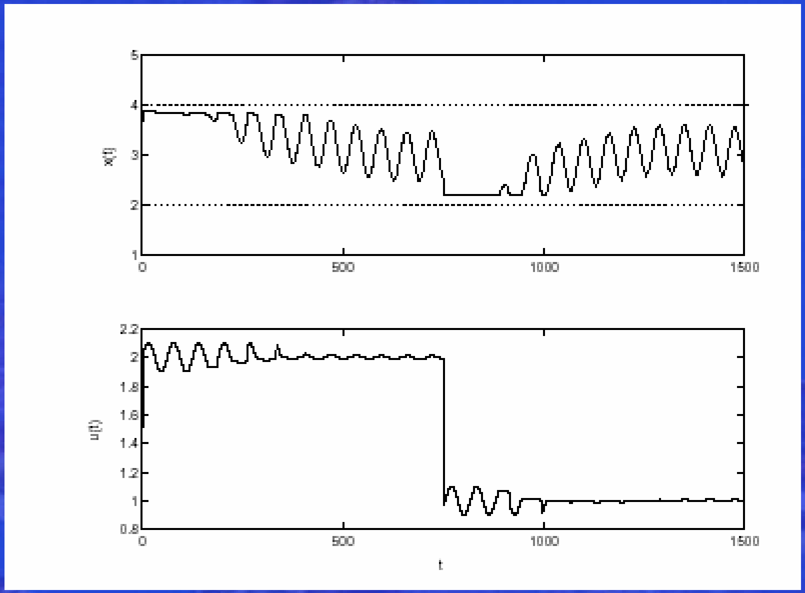
$$\dot{V} = \frac{Q_F}{V^*}(V^* - V)$$

$$\frac{d\hat{Q}_F}{dt} = \gamma(\hat{Q}_F - \overline{Q}_F)(\hat{Q}_F - \underline{Q}_F)(1 + \frac{\partial \phi}{\partial V})$$



Single tank



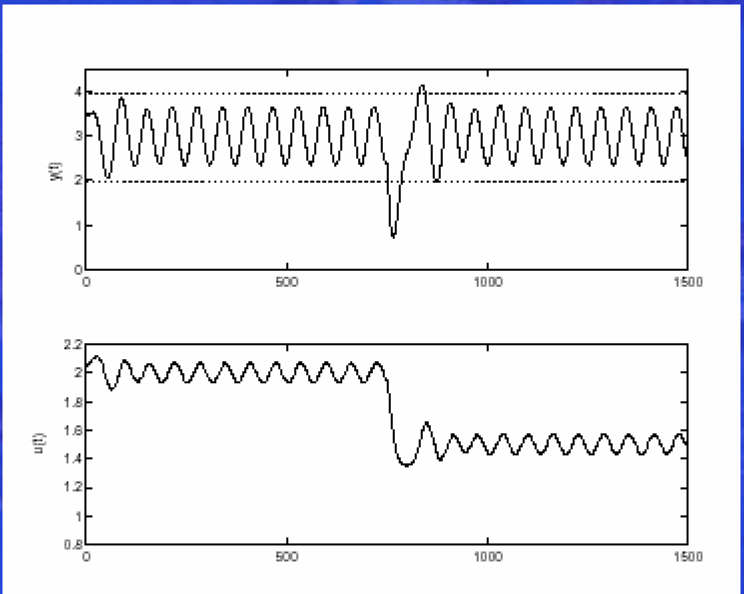


Single tank

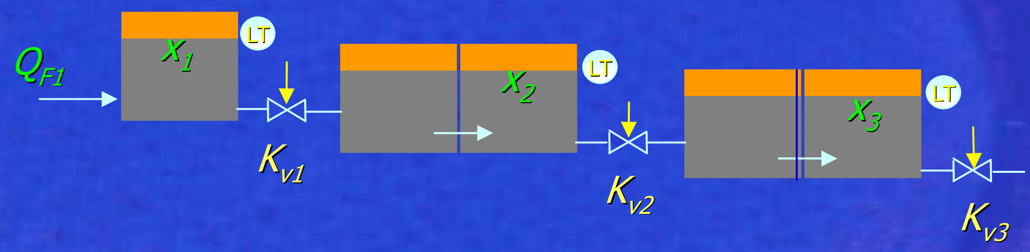
"Split range" averaging level controller

$$u = k_c \left[e + \frac{1}{T_i} \int_0^t e d\tau \right]$$

$$T_i = \begin{cases} T_{i1} & |e| < e_b \\ T_{i2} & |e| > e_b \end{cases}$$



Cascade of tanks



$$\frac{dx_i}{dt} = Q_{F_i} - Q_{o\,i}$$

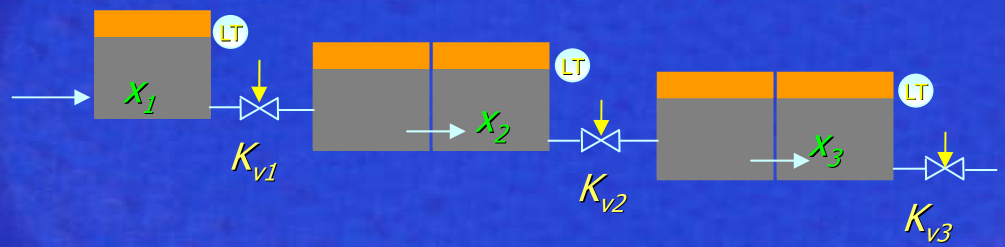
$$x_i = A_i h_i$$

$$Q_{o1} = Q_{F2} = f(K_{v_1} u_1)\sqrt{h_1 + \Delta_1 - h_2}$$

$$Q_{o2} = Q_{F3} = f(K_{v_2} u_2)\sqrt{h_2 + \Delta_2 - h_3}$$

$$Q_{o3} = f(K_{v_3} u_3)\sqrt{h_3 + \Delta_3}$$

Cascade of tanks



$$H = x_1 + x_2 + x_3 \geq 0$$

$$\dot{x} = \left(\begin{bmatrix} 0 & -\hat{u}_1 & 0 \\ \hat{u}_1 & 0 & -\hat{u}_2 \\ 0 & \hat{u}_2 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hat{u}_3 \end{bmatrix} \right) \nabla H + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} d$$

$$\hat{u}_1 = K_{v_1} u_1 \sqrt{\frac{x_1}{A_1} + \Delta_1 - \frac{x_2}{A_2}}$$

$$\hat{u}_2 = K_{v_2} u_2 \sqrt{\frac{x_2}{A_2} + \Delta_2 - \frac{x_3}{A_3}}$$

$$\hat{u}_3 = K_{v_3} u_3 \sqrt{\frac{x_3}{A_3} + \Delta_3}$$

Cascade of tanks

$$H_d(x) = H(x) + H_a(x) = \sum_{i=1}^{3} x_i + \phi_i(x_i, x_i^*, x_{\min}, x_{\max})$$

Tight control

$$\phi_i(x_i, x_i^*) = -x_i^* \ln(x_i)$$

Averaging control

$$\phi_i(x_i, x_i^*) = -x_i + \alpha \left[\frac{\left(x_i^* - \underline{x}_i\right)^2}{x_i - \underline{x}_i} - \frac{\left(\overline{x}_i - x_i^*\right)^2}{\overline{x}_i - x_i} \right]$$

Cascade of tanks

Key matching equation

$$\begin{bmatrix} 0 & -\hat{u}_1 & 0 \\ \hat{u}_1 & 0 & -\hat{u}_2 \\ 0 & \hat{u}_2 & \hat{u}_3 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H_a}{\partial x_1} \\ \dfrac{\partial H_a}{\partial x_2} \\ \dfrac{\partial H_a}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \hat{d}$$

$$\hat{u}_1 = -\hat{d} \left[\frac{\partial H_a}{\partial x_1} \right]^{-1}$$

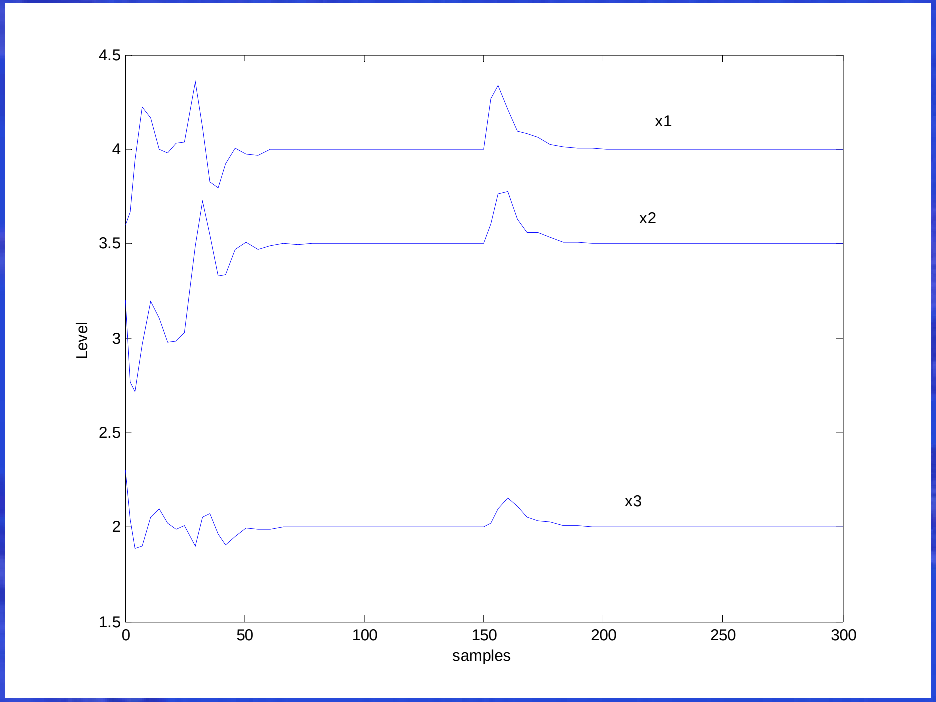
$$\hat{u}_2 = -\hat{d} \left[\frac{\partial H_a}{\partial x_1} \right] \left[\frac{\partial H_a}{\partial x_2} \frac{\partial H_a}{\partial x_3} \right]^{-1}$$

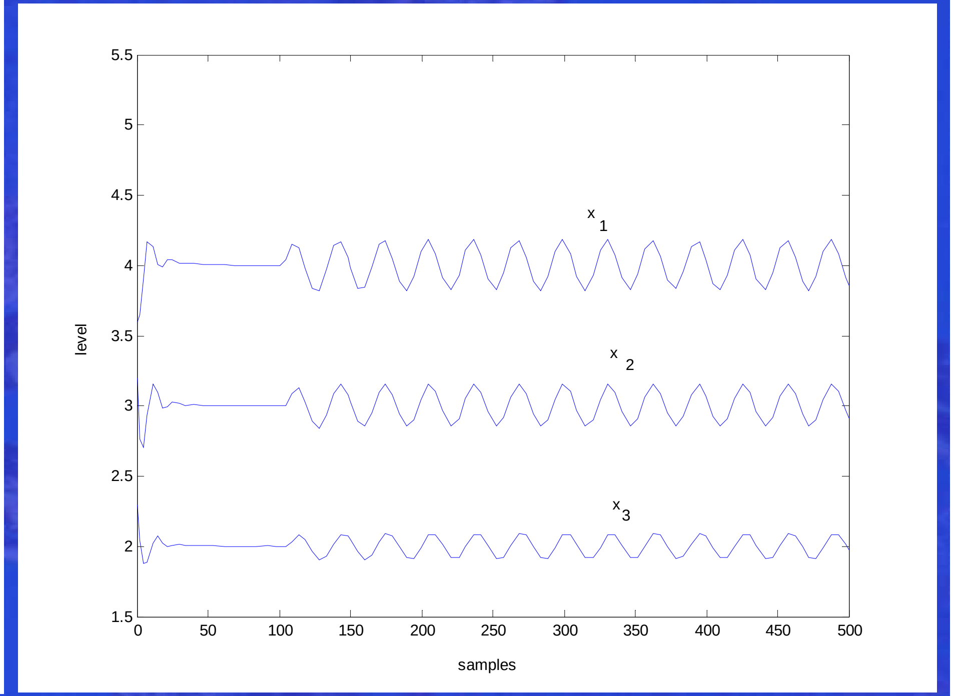
$$\hat{u}_2 = -\hat{d} \left[\frac{\partial H_a}{\partial x_1} \right] \left[\frac{\partial H_a}{\partial x_3} \right]^{-1}$$

$$\dot{\hat{d}} = \gamma (\hat{d} - \overline{d})(\hat{d} - \underline{d})(1 + \frac{\partial \phi_1}{\partial x_1})$$

Cascade of tanks

Same energy function for all cells



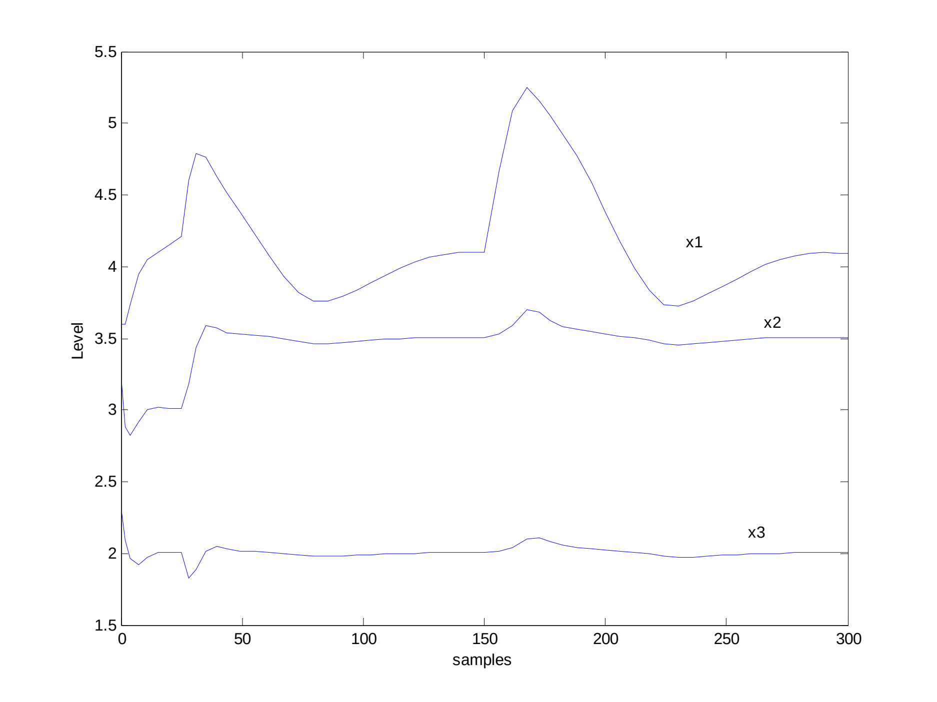


step responses

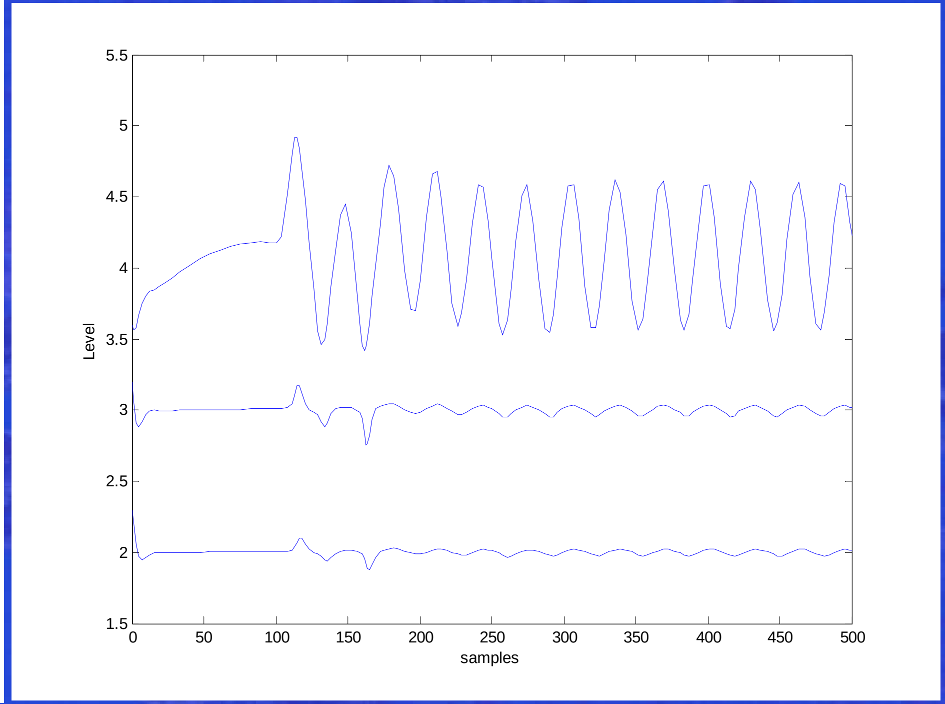
periodic disturbance

Some simulation results

First tank with averaging level strategy and same energy function for cells 2 and 3.



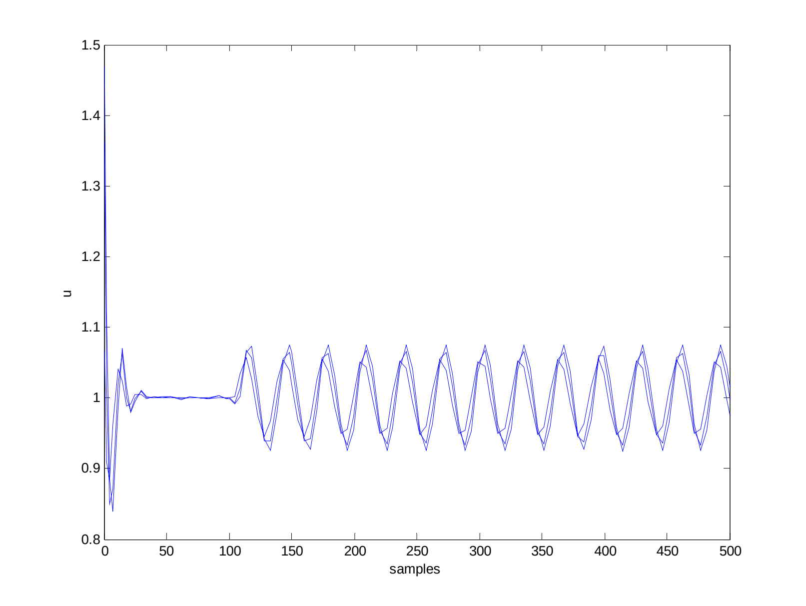
step responses



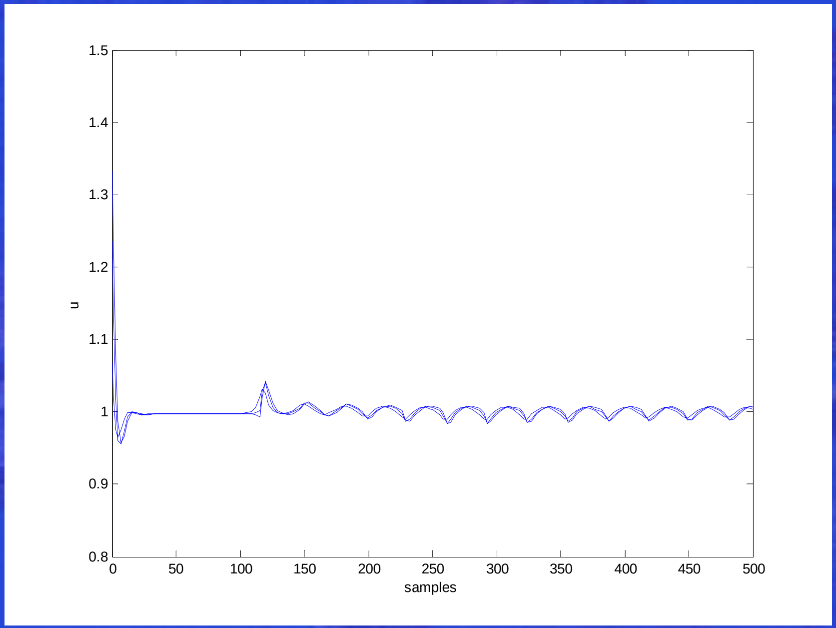
periodic disturbance

Some simulation results

Control signals – periodic disturbance



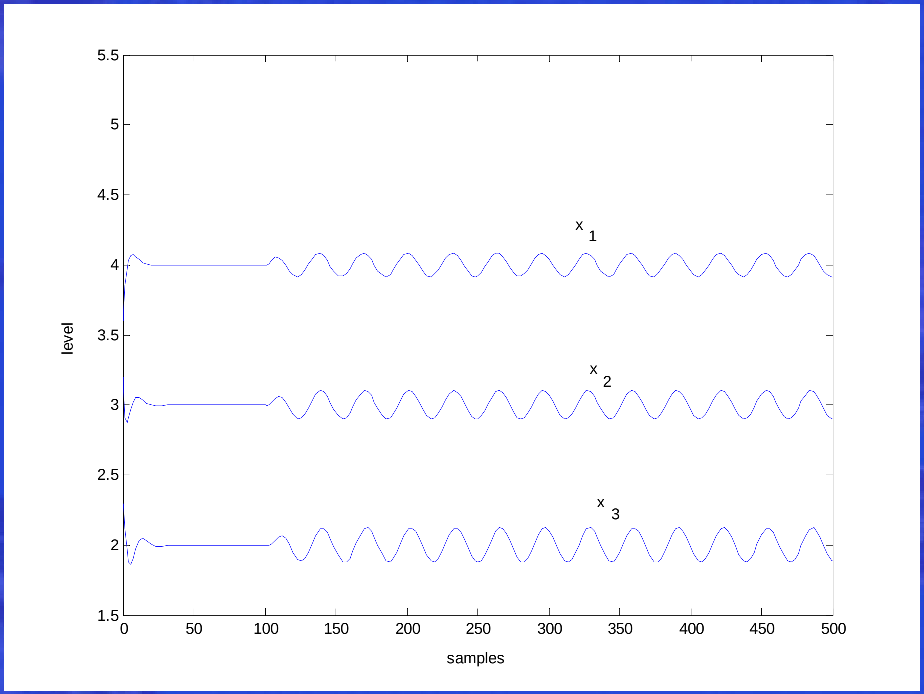
Same energy function for all cells

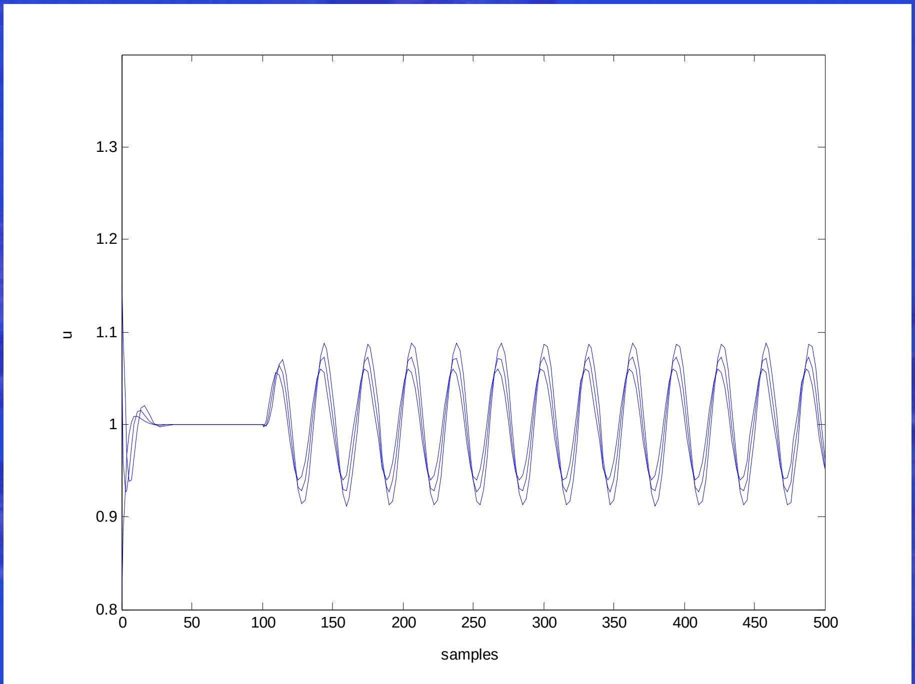


First tank with averaging level energy function

Some simulation results

Three PI controllers.





Final remarks

We have proposed a new approach based on shaping the total mass function to extend the systematic design of averaging controller to multi-tanks systems.

A simple example, considering a three tanks flotation circuit, has illustrated the main features of the proposed approach.

Future work will consider the nonlinear characteristics of the actuator, the effect of adding decoupling and damping to the controllers, as well as the presence of nonconstant disturbances.

D. Sbarbaro and R. Ortega, “Averaging level control: An approach based on mass balance”, Journal of Process Control, Volume 17, Issue 7, August 2007, Pages 621-629.