Energy based modelling and control of physical systems
Lecture 4

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References

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Outline

1. Control by interconnection

2. Interconnection and Damping Assignment - Passivity Based Control

Port-Hamiltonian control systems

Let us recall the state space model of a port-Hamiltonian control system

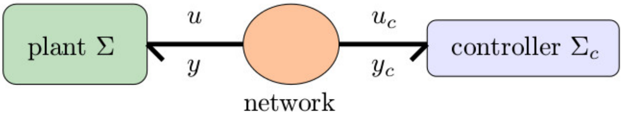
$$\dot{x} = \left(J(x) - R(x)\right)\frac{\partial H}{\partial x}(x) + g(x)u,$$
$$y = g^\top(x)\frac{\partial H}{\partial x}(x),$$

where where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$, $m < n$, is the control action, $H : \mathbb{R}^n \to \mathbb{R}$ is the total stored energy, $J(x) = -J(x)^\top$ is the $n \times n$ natural interconnection matrix, $R(x) = R(x)^\top \geq 0$ is the $n \times n$ damping matrix, $g(x)$, is the $n \times m$ input map and $\mathbf{u}, \mathbf{y} \in \mathbb{R}^m$, are conjugated variables whose product has units of power.

$$\dot{H} = u^\top y - \frac{\partial H}{\partial x}^\top R \frac{\partial H}{\partial x},$$
$$\dot{H} \leq u^\top y,$$

Control by interconnection

A controlled system may be viewed as a plant system interconnected with a control system exchanging energy

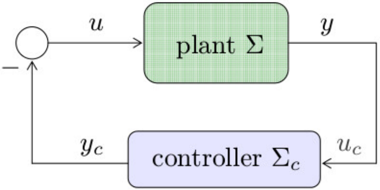


The interconnection is power continuous if

$$u^\top(t)y(t) + u_c^\top(t)y_c(t) = 0, \qquad \forall t$$

Control by interconnection

For instance: a negative feedback

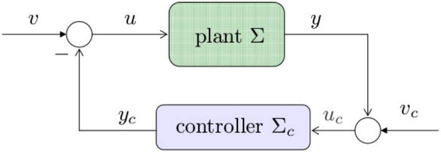


The negative feedback defines the following relation

$$\begin{matrix} u = -y_c \\ y = u_c \end{matrix} \Rightarrow u^\top y + u_c^\top y_c = -y_c^\top y_c + y^\top y_c = 0$$

Control by interconnection

Let us now consider the feedback



Now $u = v - y_c$ and $u_c = y + v_c$. Let the plant and controller have state variables x and ξ and energy functions $H(x)$ and $H(\xi)$. If the maps $u \to y$ and $u_c \to y_c$ are passive,

then the map $(v, v_c) \to (y, y_c)$ is passive with energy function $H_d(x, \xi) = H(x) + H(\xi)$.

Control by interconnection

Assume that the plant and the controller are PHS

$$\Sigma : \begin{cases} \dot{x} = [J(x) - R(x)]\dfrac{\partial H}{\partial x}(x) + g(x)u \\ y = g^\top(x)\dfrac{\partial H}{\partial x}(x) \end{cases} \quad x \in \mathcal{X}$$

$$\Sigma_c : \begin{cases} \dot{\xi} = [J_c(\xi) - R_c(\xi)]\dfrac{\partial H_c}{\partial \xi}(\xi) + g_c(\xi)u_c \\ y_c = g_c^\top(\xi)\dfrac{\partial H_c}{\partial \xi}(\xi) \end{cases} \quad \xi \in \mathcal{X}_\rfloor$$

Booth are passive systems, so a power preserving interconnection, $u = -y_c$, $y = u_c$, yields a passive closed-loop systems.

Control by interconnection

The closed-loop systems looks

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} J(x) & -g(x)g_c^\top(\xi) \\ g_c(\xi)g^\top(x) & J_c(\xi) \end{bmatrix}}_{J_{cl}(x,\xi)} - \underbrace{\begin{bmatrix} R(x) & 0 \\ 0 & R_c(\xi) \end{bmatrix}}_{R_{cl}(x,\xi)} \right) \begin{bmatrix} \frac{\partial H_d}{\partial x}(x) \\ \frac{\partial H_d}{\partial \xi}(\xi) \end{bmatrix}$$

$$\begin{bmatrix} y \\ y_c \end{bmatrix} = \underbrace{\begin{bmatrix} g(x) & 0 \\ 0 & g_c(\xi) \end{bmatrix}}_{g_{cl}} \begin{bmatrix} \frac{\partial H_d}{\partial x}(x) \\ \frac{\partial H_d}{\partial \xi}(\xi) \end{bmatrix}$$

With total energy function

$$H_d(x,\xi) = H(x) + H_c(\xi)$$

Control by interconnection

So, what now?

We would like to get an energy function in terms of x only $H_d = H_d(x)$, so that we can set the minimum at the desired point.

In order achieve this, we must restrict the dynamics to a submanifold of the (x, ξ) space parametrized by x. This means that we are looking for a submanifold

$$\Omega_C = (x, \xi) : \xi = F(x) - C$$

which is dynamically invariant, i.e.,

$$\left(\frac{\partial F_i}{\partial x}^\top \dot{x} - \dot{\xi}_i \right)_{\xi = F_i(x) - C} = 0$$

Control by interconnection

Casimir functions

Let us look for invariants that relates each state of the controller with the states of the plant:

$$C_i(x, \xi_i) = F_i(x) - \xi_i$$

In order to relate all the states of the controller with the state of the plant we define $F(x) = [F_1(x), F_2(x), \ldots, F_{n_c}(x)]$, and we obtain the following matching condition

$$\underbrace{\begin{bmatrix} \frac{\partial F}{\partial x}^\top(x) & -\mathbb{I} \end{bmatrix} \begin{bmatrix} J(x) - R(x) & -g(x)g_C^\top(\xi) \\ g_C(\xi)g^\top(x) & J_C(\xi) - R_C(\xi) \end{bmatrix}}_{\text{Matching condition}} \begin{bmatrix} \frac{\partial H_d}{\partial x}(x) \\ \frac{\partial H_d}{\partial \xi}(\xi) \end{bmatrix} = 0$$

- Only the term in blue is considered in the matching condition because we want the Casimir functions to be structural invariants of the system: not depend on $H_d(x, \xi)$.

Control by interconnection

The condition for existence of Casimir functions for the closed loop system

$$\begin{bmatrix} \frac{\partial F}{\partial x}^\top(x) & -\mathbb{I} \end{bmatrix} \begin{bmatrix} J(x) - R(x) & -g(x)g_C^\top(\xi) \\ g_C(\xi)g^\top(x) & J_C(\xi) - R_C(\xi) \end{bmatrix} = 0$$

may be written out as

Matching equations

$$\frac{\partial F}{\partial x}^\top(x)J(x)\frac{\partial F}{\partial x}(x) = J_c(\xi) \quad (1)$$

$$R(x)\frac{\partial F}{\partial x}(x) = 0 \quad (2)$$

$$R_c(\xi) = 0 \quad (3)$$

$$\frac{\partial F}{\partial x}^\top(x)J(x) = g_c(\xi)g^\top(x) \quad (4)$$

Control by interconnection

The closed-loop dynamic then takes the form

$$\dot{x} = \left[J(x) - R(x)\right]\frac{\partial H}{\partial x}(x) - g(x)g_C^\top(\xi)\frac{\partial H_c}{\partial \xi}(\xi)$$

Using the second and fourth M.C. we get

$$\dot{x} = \left[J(x) - R(x)\right]\left(\frac{\partial H}{\partial x}(x) + \frac{\partial F}{\partial x}(x)\frac{\partial H_c}{\partial \xi}(\xi)\right)$$

Since $\xi = F(x) - C$, we use the chain-rule for differentiation to establish

$$\frac{\partial F}{\partial x}(x)\frac{\partial H_c}{\partial \xi}(\xi) = \frac{\partial H_c}{\partial x}(F(x) - C)$$

Control by interconnection

Hence we obtain:

$$\dot{x} = \left[J(x) - R(x)\right] \left(\frac{\partial H}{\partial x}(x) + \frac{\partial H_c}{\partial x}(F(x) - C) \right)$$

Or equivalently

$$\dot{x} = \left[J(x) - R(x)\right] \frac{\partial H_d}{\partial x}(x)$$

With closed-loop energy $H_d(x) = H(x) + H_c(F(x) - C)$.

Control by interconnection

Let us interpret the control in terms of ***energy balancing***. Since $R_c = 0$, the energy balance equation of the controller is

$$\frac{dH_c}{dt} = u_c^\top y_c$$

Hence, along any invariant submanifold Ω_C, we have

$$\frac{dH_d}{dt} = \frac{dH}{dt} + \frac{dH_c}{dt} = \frac{dH}{dt} - u^\top y \qquad (u_c^\top y_c = -u^\top y)$$

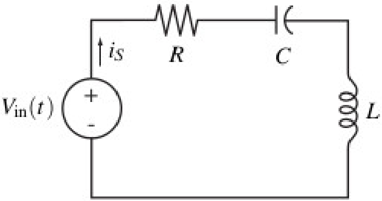
and integrating (up to some constant) we obtain

$$H_d(t) = H(t) \underbrace{- \int_0^t u^\top(\tau) y(\tau) d\tau}_{H_c}$$

We obtain the general M.E!: $H_c(t) = -\int_0^t u^\top(\tau) y(\tau) d\tau$.

Example: RLC circuit

Let us consider a simple linear RLC circuit:



Constitutive relations

$$
\begin{aligned}
u_s &= V_{in} \\
u_r &= RI_r \\
\phi &= LI_L \\
Q &= Cu_C
\end{aligned}
$$

Dynamic relations

$$u_L = \frac{d\phi}{dt}, \qquad \text{or in integral form} \qquad \phi(t) = \phi(t_0) + \int_0^t u_L(\tau)d\tau$$

$$I_C = \frac{dQ}{dt}, \qquad \text{or in integral form} \qquad Q(t) = Q(t_0) + \int_0^t I_C(\tau)d\tau$$

Example: the RLC circuit

Let us consider a RLC circuit, with dissipation and input port. The energy is

$$H(x(t)) = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{\phi^2}{L}$$

The interconnection and dissipation matrix and input vector field are

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix}, \qquad gu = \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

and the dynamic is

$$\dot{x} = (J - R)\frac{\partial H}{\partial x} + gu = (J - R)\begin{bmatrix} \frac{Q}{C} \\ \frac{\phi}{L} \end{bmatrix} + gu = \begin{bmatrix} \frac{\phi}{L} \\ -\frac{Q}{C} - R\frac{\phi}{L} + V_{in} \end{bmatrix}$$

with state variables $x = [Q, \phi]$, output $y = \frac{\phi}{L} = \frac{x_2}{L}$ and input $u = V_{in}$

- If $V_{in} = 0$, the natural equilibrium is $x = (0, 0)$. If on other hand $V_{in} = V^*$, the forced equilibrium point is $x = (x_1^*, 0)$, with $x_1^* = CV^*$.

Example: RLC circuit

Let us synthesis the controller using the M.C.s.

- From physical considerations we know that we only need to shape the x_1 coordinate: F is only one scalar function.
- From M.E.2 we obtain that $\frac{\partial F}{\partial x_2} = 0$.
- Then, from M.E.1. we obtain that $J_c = 0$, and from M.E.3 that $R_c = 0$.
- Finally from M.E.4 we have that $\frac{\partial F}{\partial x_1} = g_c(\xi)$ and that $\xi \in \mathbb{R}$.

We would like to have

$$H_c(x_1) = \tfrac{1}{2C_a}x_1^2 - \left(\tfrac{1}{C_a} + \tfrac{1}{C}\right) x_1 x_1^*, \quad \text{such that} \quad \boxed{H_d(x) = \frac{1}{2}\frac{(x_1 - x_1^*)^2}{(C + C_a)} + \frac{1}{2}\frac{x_2}{L}^2}$$

This is achieved if we select $F(x_1) = x_1$ and $C = 0$ on the invariant submanifold such that $\xi = x_1$. The control system is then given by (using condition $\frac{\partial F}{\partial x_1} = g_c(\xi)$)

$$\dot{\xi} = u_c$$

$$y_c = \frac{\partial H_c}{\partial \xi}$$

Control by interconnection

Remarks

- The port-Hamiltonian structure provide important information for finding the solutions of the control system,
- The control has physical interpretation in terms of interconnection and energy balancing
- The Casimir method can be used to analyse new stability profiles of interconnected systems

Outline

1. Control by interconnection

2. Interconnection and Damping Assignment - Passivity Based Control

IDA–PBC

IDA-PBC objective

Find a static state-feedback control $u(x) = \beta(x)$ such that the closed-loop dynamics is a PH system with dissipation of the form

$$\dot{x} = (J_d - R_d)\frac{\partial H_d}{\partial x},$$

$H_d(x)$, has a strict local minimum at x^*,
$J_d(x, u) = -J_d(x, u)^T$,*desired* interconnection matrix,
$R_d(x, u) = R_d(x, u)^T \geq 0$, *desired* dissipation matrix,

IDA-PBC

Suppose that the following PDE is verified

$$g^{\perp}(J-R)\frac{\partial H}{\partial x} = g^{\perp}(J_d - R_d)\frac{\partial H_d}{\partial x}$$

where $g^{\perp}(x)g(x) = 0$ y $H_d(x)$ is such that

$$\frac{\partial H_d}{\partial x}(x^*) = 0, \qquad \frac{\partial^2 H_d}{\partial x^2}(x^*) > 0,$$

then there exists a control $u = \beta(x)$

Defined as $\beta = (g^{\top}g)^{-1}g^{\top}[(J_d - R_d)\frac{\partial H_d}{\partial x} - (J-R)\frac{\partial H}{\partial x}]$ such that the closed-loop system takes the PH form

$$\dot{x} = (J_d - R_d)\frac{\partial H_d}{\partial x},$$

And closed-loop energy

$$\dot{H}_d = -\frac{\partial H_d^{\top}}{\partial x} R_d \frac{\partial H_d}{\partial x} < 0, \ \forall x \neq x^* \quad \text{and} \quad \dot{H}_d(x^*) = 0.$$

Remarks

Degrees of freedom in the design

- J_d and R_d are free–up to the constraint of skew–symmetry and positive semidefiniteness, respectively.
- H_d may be totally, or partially fixed, provided we can ensure $\frac{\partial H_d}{\partial x}(x^*) = 0$, $\frac{\partial^2 H_d}{\partial x^2}(x^*) \geq 0$ and probably a properness condition.
- there is an additional degree of freedom in $g^{\perp}(x)$ which is not uniquely defined by $g(x)$.

Attention: Requires the solution of a quasilinear PDE

the method of characteristics...

Example:

Part of the homework...

Final remarks

- Energy-based methods permit to study systems from the universal perspective of energy transfer,
- Physically coherent models and controllers,
- Defines a class of non-linear systems
- The method are applicable to different physical domains and state representations (infinite dimensions, discrete, etc..)

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Muchas gracias!