EM-based channel estimation in OFDM systems with phase noise

Rodrigo Carvajal, Juan C. Agüero, Boris I. Godoy, Graham C. Goodwin.

Centre for Complex Dynamic Systems and Control (CDSC)
The University *of* Newcastle, Australia.

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Outline

Problem of interest

- General model with phase noise:

$$\phi_{k+1} = \phi_k + v_{k+1},$$
$$\mathbf{x}_{k+1} = \mathbf{x}_k,$$
$$\mathbf{r}_k = e^{j\phi_k}(\mathbf{e}_k^T \tilde{\mathbf{H}})\mathbf{x} + \eta_k, \quad \eta_k \sim \mathrm{N}(0, \sigma_\eta^2)$$

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- We propose to solve the estimation problem using Maximum likelihood.
- Difficulties:
  - Clearly define parameters and variables.
  - Hidden variables $\mathbf{z}$, i.e. $p(\mathbf{r}|\theta) = \int p(\mathbf{r},\mathbf{z}|\theta)d\mathbf{z}$.

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Introduction

- The EM algorithm has previously utilized in OFDM systems parameter estimation[1]
- The estimation of multiple parameters in OFDM systems with phase noise has been addressed previously in the literature[2]

[1] R. Mo, et. al, "An EM-based semiblind joint channel and frequency offset estimator for OFDM systems over frequency selective fading channels," *IEEE Trans. Veh. Technol.,* vol. 57, no. 5, pp.3275–3282, Sep. 2008.

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Introduction

- → In our approach, the channel is regarded as a constant parameter.
- → We exploit the linear and Gaussian structure associated with the transmitted signal.
- → We obtain expressions that consider different levels of training.
- → We study the joint estimation of PHN bandwidth and CIR, and the effect that the associated estimation errors have on the CIR.
- → We show that inaccurate PHN bandwidth estimation introduces errors in the estimation of the CIR when the number of subcarriers is low.

[1] R. Mo, et. al, "An EM-based semiblind joint channel and frequency offset estimator for OFDM systems over frequency selective fading channels," *IEEE Trans. Veh. Technol.,* vol. 57, no. 5, pp.3275–3282, Sep. 2008.

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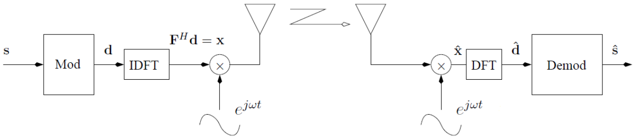

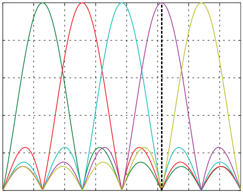

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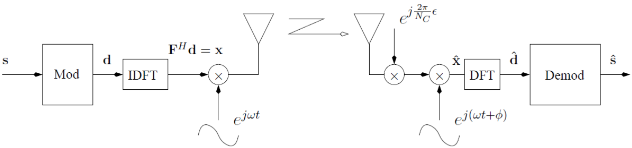

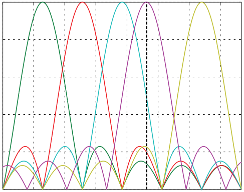

Outline

System Model

Phase Noise

→ In *free-running* oscillators, PHN is modelled as a continuous Brownian motion process:

$$\phi_{k+1} = \phi_k + v_{k+1},$$

where:

- v_k: i.i.d zero-mean Gaussian variable with variance $\sigma_v^2 = 2\pi\beta T/N_C$.
- β: PHN bandwidth.
- T symbol duration ($1/T$ is the symbol rate).

Carrier Frequency Offset

→ CFO can be modelled as a diagonal matrix $\mathbf{C}_\varepsilon = e^{j\text{diag}\left(\frac{2\pi\varepsilon k}{N_C}\right)}$, with $k = 0, 1, \ldots, N_C - 1$.

→ ε is the *normalized* frequency offset ($|\varepsilon| \leq 1/2$).

System Model

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- The transmitted signal $\mathbf{x}$ can be expressed as a linear combination of a deterministic signal (known) and a stochastic signal (unkwnown), as

$$\mathbf{x} = (\bar{\mathbf{x}}_k + \tilde{\mathbf{x}}).$$

- Then, we have:

$$\begin{aligned} \phi_{k+1} &= \phi_k + v_{k+1}, \quad v_k \sim \mathrm{N}(0, 2\pi\beta T/N_C) \\ \bar{\mathbf{x}}_{k+1} &= \bar{\mathbf{x}}_k, \\ \mathbf{r}_k &= e^{j\psi_k}(\mathbf{e}_k^T \tilde{\mathbf{H}})(\bar{\mathbf{x}}_k + \tilde{\mathbf{x}}) + \eta_k, \quad \eta_k \sim \mathrm{N}(0, \sigma_\eta^2) \end{aligned}$$

where

  - $\tilde{\mathbf{H}}$ is the (circulant) channel matrix,
  - $\psi_k = \phi_k + \frac{2\pi k \varepsilon}{N_C}$,
  - $\bar{\mathbf{x}}_k$ is the (unknown) stochastic part of $\mathbf{x}$,
  - $\tilde{\mathbf{x}}$ is the (known) training part of $\mathbf{x}$,
  - and $\mathbf{e}_k$ is the kth column of the identity matrix.

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- The estimation problem requires the maximization of

$$\hat{\theta} = \arg\max_{\theta} \, l(\theta),$$

where

- $\theta = [\mathbf{h}^T, (\beta T)^{-1}]$,
- $l(\theta) = \log\{p(\mathbf{r}\,|\,\theta)\}$,
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Difficulties

$\rightarrow$ The system is nonlinear (due to the exponential $e^{j\phi_k}$).

$\rightarrow$ There is no measurement of the phase noise, $\phi_{0:N_C-1}$.

The Expectation-Maximization (EM) algorithm

- EM is an iterative algorithm to obtain the ML estimate[3].
- Unknown signals are treated as *hidden variables*[3].

[3] A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *J. R. Stat. Soc. B,* vol. 39, no. 1, pp. 1–38, 1977.

[4] T. Schön, F. Gustaffson, and P.-J. Nordlund, "Marginalized particle filters for mixed linear/nonlinear state-space models," *IEEE Trans. Signal Process.,* vol. 53, no. 7, pp. 2279–2289, July 2005.

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E-step: Compute the expected value

$$Q(\theta, \hat{\theta}_k) = E\{\log[p(\bar{\mathbf{x}}, \phi, \mathbf{r}|\theta)]|\mathbf{r}, \hat{\theta}_k\}$$

M-step: Obtain a new estimate by maximizing the function Q, as

$$\hat{\theta}_{k+1} = \arg\max_{\theta} Q(\theta, \hat{\theta}_k)$$

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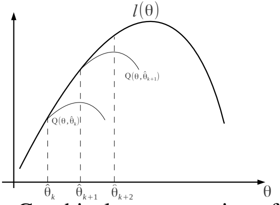

Graphical representation of the EM algorithm

The implementation of the EM algorithm involves the computation of the probability density function $p(\bar{\mathbf{x}}, \phi|\mathbf{r}, \theta)$.

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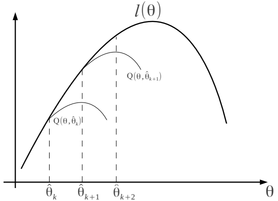

Graphical representation of the EM algorithm

We obtain $p(\bar{\mathbf{x}}, \phi|\mathbf{r}, \theta) = \underbrace{p(\bar{\mathbf{x}}|\phi, \mathbf{r}, \theta)}_{\text{Kalman filter}} \underbrace{p(\phi|\mathbf{r}, \theta)}_{\text{Particle filter}}$, by utilizing the Marginalized particle filter[4].

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Numerical Examples

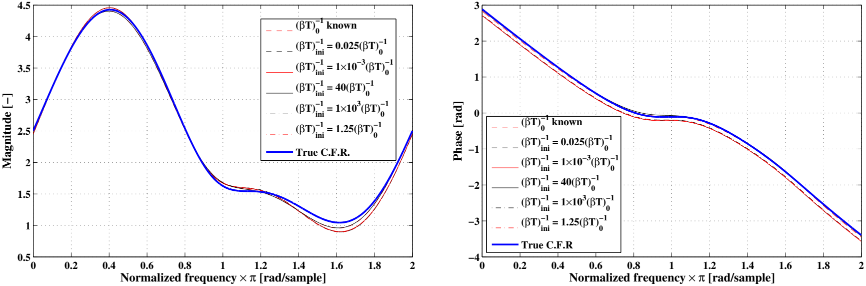
Simulation setup:

- The signal $\mathbf{x}$ is considered known (*training*).
- A Rayleigh fading channel with $L = 4$ taps, $(\beta T)^{-1} = 1000$ $(\beta T = 0.001)$
- Known $\varepsilon = 0.2537$, $\sigma_\eta^2 = 0.01$, and $\text{SNR} = 10[\text{dB}]$.
- The number of *particles* used in the *particle smoother* is 100 (constant), and the number of iterations of the EM algorithm is 150.
- The vector parameter to estimate is $\theta = [\mathbf{h}^T \, (\beta T)^{-1}]^T$.
- In a recent paper, it was shown that the PHN bandwidth cannot be accurately estimated[5]. Hence, several initial guess are considered for $(\beta T)^{-1}$.

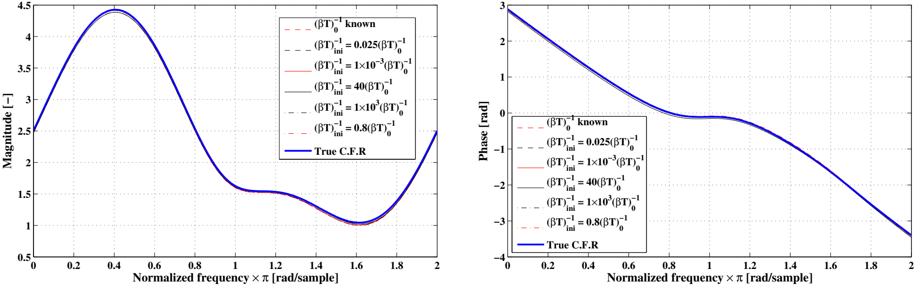
[5] R. Carvajal, J. C. Agüero, B. I. Godoy, and G. C. Goodwin, "On the accuracy of phase noise bandwidth estimation in OFDM systems," in *Proc. IEEE Int. Work. Signal Process. Adv. Wireless Commun. (SPAWC)* pp. 246–250, San Francisco, USA, 26–29 June 2011.

Numerical Examples

$N_C = 64$



Frequency response of the estimated channel with different initial guesses for the PHN bandwidth. $N_C = 64$, $(\beta T)_0^{-1} = 1000$.



Frequency response of the estimated channel with different initial guesses for the PHN bandwidth. $N_C = 256$, $(\beta T)_0^{-1} = 1000$.

Outline

Extensions of our work

Submitted to *IEEE Trans. on Veh. Technol.*

Extension to the estimation of CFO, ε, and channel noise variance, σ_η^2.

Extension to proper and improper signals (for different modulation schemes, such as BPSK, GMSK).

Analysis of the impact of different training levels on the overall parameter estimation.

Analytic expression for Cramer Rao Lower Bound of phase noise bandwidth (expressed as $(\beta T)^{-1}$).

Outline

Conclusions

- → We have exploited the linear and Gaussian structure associated with the transmitted signal.
- → *Maximum likelihood* estimation of the channel impulse response (CIR) can be successfully performed in OFDM systems, based on *particle smoothing* and the EM algorithm.
- → The impact of the inaccurate PHN bandwidth estimation on the estimation of CIR is negligible when the number of subcarriers is relatively high (e.g. 256 or more).
- → For a small number of subcarriers, inaccurate estimation of CIR can potentially have a significant impact on the estimation of the received signal and hence lead to an increase in the bit error rate.

Questions ?