Upper bounds on the best achievable performance subject to SNR constraints in two-channel networked architectures.

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Abstract

This paper studies control problems subject to signal-to-noise ratio (SNR) constraints for two-channel MIMO architectures. We first establish a second order moments equivalence between LTI feedback systems subject to SNR constraints and a class of Markov jump linear systems (MJLSs). This equivalence allows one to recast the original SNR constrained problem into an optimal mode-independent controller design problem for MJLSs. Exploiting this fact, and provided some linear matrix inequalities are feasible, we compute an upper bound on the best achievable performance subject to SNR constraints in two-channel architectures, when static state feedback controllers are employed. A numerical example is included to illustrate the results.

I. Introduction

Contemporary control systems often integrate sensors, actuators and controllers over constrained communication networks. Such systems are called Networked Control Systems (NCSs) [1]. The NCS research area focuses mainly on studying the interplay between control objectives and communication constraints. The most commonly studied communication constraints include data-rate limits, random delays, data-dropouts, and SNR constraints [2]. There exists a vast amount of literature addressing the associated control problems; see, e.g., [3,4,2,5–7] and the references therein. However, no unified treatment of networked control problems has emerged yet. In this paper we focus on SNR constraints.

The study of SNR constraints in networked control was initiated by [5], where the problem of mean square stabilization was addressed. The main conclusion of [5] is that an LTI plant can be stabilized if and only if the available SNR is larger than a function of the unstable plant poles. That work has been extended in several ways in, e.g., [8–13], where also performance related questions have been addressed. All the work referred to above considers situations where only one single-input single-output SNR constrained channel is present. Given the fact NCSs usually comprise a large number of interacting nodes, it is thus relevant to look at the multi-channel case. As a first step in that direction we consider in this paper a setup where two multiple-input multiple-output channels are employed.

The first contribution of this paper lies in noting that the optimal LTI controller design problem for the considered two-channel architecture, is equivalent to designing controllers for a class of Markov Jump Linear Systems (MJLS; [14]). We arrive at this conclusion by extending the results in [15] to show that, not only when one single-input single-output channel is present, but also for situations that employ multiple channels as well, there exists a second order moments equivalence between SNR constrained LTI systems and a class of MJLSs. In addition, we show in this paper that the equivalence is actually instantaneous and not only stationary (cf. [15]). The relevance of this equivalence is two-fold. First, it allows one to solve control problems subject to data dropouts using SNR related results (e.g., [5, 13]). This approach is illustrated in [15] for an NCS closed over a single-input single-output erasure channel. Second, it allows one to solve control problems subject to SNR constraints by using results in the literature on control over erasure channels (see, e.g., [4] and the references therein), or the results of MJLS theory [14]. We adopt the latter approach in this paper.

When designing controllers for MJLS, a crucial assumption relates to whether or not the controller has instantaneous access to the state of the associated Markov chain. If that is the case, then the controller is said to be mode-dependent. If not, then the controller is said to be mode-independent. Most of the literature on MJLSs focuses on the mode-dependent case [14, 16]. This assumption is usually not valid in networked situations because packet acknowledgements, if available, are usually only available with some delay (e.g., TCP-like protocols; [4]). Moreover, in this paper, the MJLS setting is just an auxiliary situation and there exists no clear interpretation of what measuring the Markov chain state means from the point of view of the original SNR constrained control problem. We thus conclude that the results in [14, 16, 17], where mode-dependent feedback is studied, cannot be applied to the class of problems of interest here. It is however possible to consider mode-independent control strategies for MJLS, but at the expense of using static control laws [17–19]. For simplicity, we will build upon [19] and thus we will focus on static state feedback control laws. As the reader will certainly notice, our approach can be readily extended to any situation where the corresponding mode-independent MJLS design problem can be solved.

By using the results mentioned above, we are able to compute an upper bound on the best achievable performance in two-channel SNR constrained control architectures, when the controllers are constrained to be of the static state feedback

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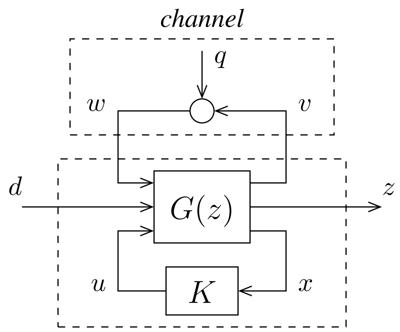

Fig. 1. Networked control system closed over an additive noise channel.

type. This bound is provided in terms of a convex optimization problem involving LMIs [20], and constitutes the second contribution of the paper.

The remainder of this paper is organized as follow: Section II presents the notation used through this paper. Section III states the problem of interest and our working assumptions. Section IV presents the main result of the paper, namely an instantaneous second order moments equivalence between LTI systems subject to SNR constraints and a class of MJLSs. Section V shows how to use the results in Section IV to give bounds on the solution of the optimal control problem stated in Section III. Section VI presents a simulation study, whilst Section VII draws conclusions.

II. Notation

$\mathbb{R}$ and $\mathbb{N}$ refer the real and natural numbers, respectively. $\mathbb{N}_0 \triangleq \mathbb{N}\cup\{0\}$, $\mathbb{R}^+ \triangleq \{x \in \mathbb{R} : 0 < x < \infty\}$ and $\mathbb{R}_0^+ \triangleq \mathbb{R}^+\cup\{0\}$. $\mathcal{P}\{*\}$ stands for the probability of $(*)$ and $\mathcal{E}\{*\}$ denotes the expectation of $(*)$. Given a matrix W, W^T and W^H denote its transpose and conjugate transpose, respectively. For a sequence $\{w(k); k \in \mathbb{N}_0\}$, we define $\|w\|^2 \triangleq \sum_{k=0}^{\infty} w(k)^T w(k)$. I_n denotes the $n \times n$ identity matrix, and $0_n \triangleq 0\, I_n$. $\operatorname{diag}\{\cdot\}$ constructs block diagonal matrix from its arguments. We use z as the argument of the z-transform and also as the forward shift operator, where the particular meaning will be clear from the context.

In this paper, all random process are real valued and defined for $k \in \mathbb{N}_0$. We write x as shorthand for $\{x(k); k \in \mathbb{N}_0\}$. For any process x we define: $\mu_x(k) \triangleq \mathcal{E}\{x(k)\}$, $P_x(k) \triangleq \mathcal{E}\left\{(x(k)-\mu_x(k))(x(k)-\mu_x(k))^T\right\}$, $R_x(k+\tau,k) \triangleq \mathcal{E}\left\{(x(k+\tau)-\mu_x(k+\tau))(x(k)-\mu_x(k))^T\right\}$, $\sigma_x^2(k) \triangleq \operatorname{trace}\{P_x(k)\}$. We refer to $P_x(k)$ as the covariance matrix of x, and to $\sigma_x^2(k)$ as the variance of x. We also define (when the limits exist) $P_x \triangleq \lim_{k\to\infty} P_x(k)$ and $\sigma_x^2 \triangleq \lim_{k\to\infty} \sigma_x^2(k)$. P_x is the stationary covariance matrix of x, and σ_x^2 is the stationary variance of x. In addition, if x is a wide sense stationary (wss) (asymptotically wss) process, then $S_x\left(e^{j\omega}\right)$ denotes its (stationary) power spectral density (PSD) and $\Omega_x(e^{j\omega})$ denotes any spectral factor of $S_x\left(e^{j\omega}\right)$, i.e., $\Omega_x(e^{j\omega})\Omega_x(e^{j\omega})^H \triangleq S_x(e^{j\omega})$. We say that a random variable (process) is a second order one if and only if it has finite mean and finite second order moments (for all time instants $k \in \mathbb{N}_0$ and also when $k \to \infty$).

III. Problem Definition

This paper focuses on the NCS of Fig. 1, where G is an LTI plant whose state x is available for measurement, u is the control input, d is a disturbance, z is a signal related to performance (i.e., a controlled output), and K is a static state feedback gain. The system of Fig. 1 also includes a non-transparent channel as the link between the signals v and w.

We consider a two-channel architecture where the channel input v is split in two signals v_1 and v_2. Each of these signals is transmitted through an additive white noise (AWN) channel subject to a stationary input variance constraint (see e.g. [13, 5, 21]). In practice, it is sensible to consider pre- and post-scaling factors around AWN channels. By doing so, SNR constrained AWN channels arise (see details in Section 3 of [13]). The following definition, extended from [13] to the two-channel case, condenses the relevant features of such channels:

***Definition** 1 (Two-block SNR constrained AWN channel):* The channel in Fig. 1, with input $v \triangleq [v_1^T \;\; v_2^T]^T$ and output $w \triangleq [w_1^T \;\; w_2^T]^T$ ($v_i(k), w_i(k) \in \mathbb{R}^{n_i}$, $i \in \{1,2\}$), is a two-block SNR constrained AWN channel if and only if, $\forall\, k \in \mathbb{N}_0$, $\forall\, v(k) \in \mathbb{R}^{n_1+n_2}$,

$$w(k) = q(k) + v(k)\,, \quad q \triangleq \left[q_1^T \;\; q_2^T\right]^T\,, \tag{1}$$

where q_1 and q_2 ($q_i(k) \in \mathbb{R}^{n_i}$, $i \in \{1,2\}$) are uncorrelated zero mean white noise sequences with covariance matrices P_{q_i} that are design variables in the set of positive semidefine matrices in $\mathbb{R}^{n_i \times n_i}$, that are to be chosen subject to the constraint

$$P_{v_i} \leq \Gamma_i P_{q_i}\,. \tag{2}$$

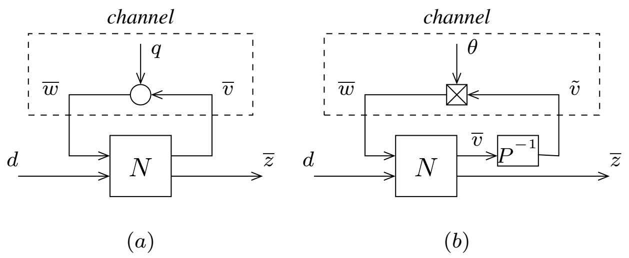

Fig. 2. System N (a) over an AWN channel, and (b) over an analog erasure channel.

In (2), P_{v_i} is the stationary variance of v_i (assumed to exist), and $\Gamma_i \in \mathbb{R}^+$ is the maximum admissible SNR for channel i. □

We assume that the plant G has the state-space description

$$\begin{bmatrix} x(k+1) \\ z(k) \\ v(k) \end{bmatrix} = \begin{bmatrix} A_g & B_u & B_d & B_w \\ C_z & D_z & D_{dz} & D_{wz} \\ C_v & D_v & D_{dv} & D_{wv} \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \\ d(k) \\ w(k) \end{bmatrix}, \tag{3}$$

$k \in \mathbb{N}_0$, $x(0) = x_0$, where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, $d(k) \in \mathbb{R}^p$, $z(k) \in \mathbb{R}^r$, and $v(k)$ and $w(k)$ are as in Definition 1. We will work under the following assumptions:

Assumption 1:

a) x_0 is a zero mean second order random variable, with mean μ_0 and covariance matrix P_0.
b) The disturbance d is a zero mean second order white noise sequence, uncorrelated with x_0, and with covariance matrix $P_d = I_p$.
c) The channel noise q is uncorrelated with (d, x_o).
d) $D_{wv} = 0$.
e) $D_{dz} = 0$ and $D_{dv} = 0$. □

As foreshadowed above, we assume that the state of G can be measured. In particular, we focus on finding the static state-feedback control law $u = Kx$ that minimizes the stationary variance of the signal z, subject to the stationary SNR constraints in (2). More precisely, we aim at solving the following problem:

Problem 1: Consider the NCS of Fig. 1, where G has the realization in (3), the channel is a two-block SNR constrained AWN channel and Assumption 1 holds. For maximum allowable SNRs $\Gamma_i \in \mathbb{R}^+$, $i = \{1, 2\}$, find

$$\left[\sigma_z^2\right]_{\Gamma_1,\Gamma_2} \triangleq \inf_{\substack{K \in \mathcal{S}^{\mathcal{L}},\ 0 \le P_{q_i} < \infty \\ P_{v_i} \le \Gamma_i P_{q_i}}} \sigma_z^2$$

where $\mathcal{S}^{\mathcal{L}} \triangleq \{K \in \mathbb{R}^{m\times n}$: the closed loop of Fig. 1 is internally stable$\}$, and σ_z^2 is the stationary variance of z. □

We note that, consistent with Definition 1, Problem 1 considers that both the noise variances P_{q_i} and the static state feedback gain K are decision variables.

IV. Equivalences

As a first step towards addressing Problem 1, we show in this section that there exists a second order moment equivalence between SNR constrained LTI systems and a class of MJLS systems. This key result will be used in Section V to show that solving Problem 1 is equivalent to solving an optimal control problem for a certain MJLS.

Consider the general networked setup of Fig. 2(a). In that figure, the channel is a two-block SNR constrained AWN channel with input $\bar{v}$ and output $\bar{w}$, d is an exogenous signal, and N is an LTI system with state-space description

$$\begin{bmatrix} \overline{x}(k+1) \\ \overline{z}(k) \\ \overline{v}(k) \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B}_d & \overline{B}_{\overline{w}} \\ \overline{C}_{\overline{z}} & \overline{D}_{d\overline{z}} & \overline{D}_{\overline{wz}} \\ \overline{C}_{\overline{v}} & \overline{D}_{d\overline{v}} & \overline{D}_{\overline{wv}} \end{bmatrix} \begin{bmatrix} \overline{x}(k) \\ d(k) \\ \overline{w}(k) \end{bmatrix}, \quad \overline{x}(0) = \overline{x}_0\,, \tag{4}$$

where $\overline{x} \in \mathbb{R}^{\overline{n}}$. A state space description of the closed loop LTI system of Fig. 2(a) is thus given by

$$\overline{x}(k+1) = A_p\overline{x}(k) + B_p d(k) + \overline{B}_{\overline{w}} q(k), \tag{5a}$$

$$\overline{z}(k) = C_p\overline{x}(k) + D_p d(k) + \overline{D}_{\overline{wz}} q(k), \tag{5b}$$

with $\overline{x}(0) = \overline{x}_0$ and

$$A_p \triangleq \overline{A} + \overline{B}_{\overline{w}}\overline{C}_{\overline{v}}\,, \quad B_p \triangleq \overline{B}_d + \overline{B}_{\overline{w}}\overline{D}_{d\overline{v}}\,,$$

$$C_p \triangleq \overline{C}_{\overline{z}} + \overline{D}_{\overline{w}\overline{z}}\overline{C}_{\overline{v}}\,, \quad D_p \triangleq \overline{D}_{d\overline{z}} + \overline{D}_{\overline{w}\overline{z}}\overline{D}_{d\overline{v}}\,.$$

For future reference we note that the use of a two-block channel between $\bar{v}$ and $\bar{w}$ introduces the partitions $\overline{v} \triangleq [\overline{v}_1^T \;\; \overline{v}_2^T]^T$ and $\overline{w} \triangleq [\overline{w}_1^T \;\; \overline{w}_2^T]^T$, with $\bar{v}_i(k), \bar{w}_i(k) \in \mathbb{R}^{n_i}$, and also the partitions

$$\overline{B}_{\overline{w}} \triangleq \begin{bmatrix}\overline{B}_{\overline{w}_1} & \overline{B}_{\overline{w}_2}\end{bmatrix}, \quad \overline{C}_{\overline{v}} \triangleq \begin{bmatrix}\overline{C}_{\overline{v}_1}^T & \overline{C}_{\overline{v}_2}^T\end{bmatrix}^T, \quad \overline{D}_{d\overline{v}} \triangleq \begin{bmatrix}\overline{D}_{d\overline{v}_1}^T & \overline{D}_{d\overline{v}_2}^T\end{bmatrix}^T. \tag{6}$$

Through this section we assume that the following holds:

***Assumption** 2:*

a) $\overline{x}_0$ is a second order random variable with mean $\overline{\mu}_0$ and covariance matrix $\overline{P}_0$.
b) The disturbance d is a zero mean second order white noise sequence, uncorrelated with $\overline{x}_0$, and with covariance matrix $P_d = I_p$.
c) The channel noise q is uncorrelated with $(d, \bar{x}_o)$.
d) $\overline{D}_{\overline{w}\overline{v}} = 0$. □

Consider now the auxiliary situation of Fig. 2(b). In that figure we have replaced the two-block SNR constrained AWN channel linking $\bar{v}$ and $\bar{w}$ in Fig. 2(a) by a channel such that, $\forall\, k \in \mathbb{N}_0$, $\forall\, \bar{v}(k) \in \mathbb{R}^{n_1+n_2}$,

$$\bar{w} = \theta(k)P^{-1}\bar{v}(k), \tag{7}$$

where

$$P^{-1} \triangleq \operatorname{diag}\left\{\Gamma_1^{-1}(1+\Gamma_1)I_{n_1}, \Gamma_2^{-1}(1+\Gamma_2)I_{n_2}\right\}, \tag{8}$$

$$\theta(k) \triangleq \operatorname{diag}\left\{\theta_1(k)I_{n_1}, \theta_2(k)I_{n_2}\right\}, \tag{9}$$

and $\theta_i(k) \in \{0,1\}$, $i \in \{1,2\}$, models data dropouts. The processes θ_1, θ_2 are assumed to satisfy the following:

***Assumption** 3:* The process θ_i, $i \in \{1,2\}$, is a sequence of i.i.d. Bernoulli random variables such that $\mathcal{P}\left\{\theta_i(0) = 1\right\} = p_i^o$, and $\mathcal{P}\left\{\theta_i(k) = 1\right\} = p_i \triangleq \Gamma_i(1+\Gamma_i)^{-1}$ for $k \geq 1$, with $0 < p_i^o < 1$. Moreover, θ_1 is independent of θ_2, and θ_i is independent of $(\overline{x}_0, d)$. □

Given (4) and (7)-(9), a state-space description of the system of Fig. 2(b) is given by

$$\overline{x}(k+1) = A\left(\theta(k)\right)\overline{x}(k) + B\left(\theta(k)\right)d(k), \tag{10a}$$

$$\overline{z}(k) = C\left(\theta(k)\right)\overline{x}(k) + D\left(\theta(k)\right)d(k), \tag{10b}$$

with $\overline{x}(0) = \overline{x}_0$ and

$$\begin{aligned}
A\left(\theta(k)\right) &\triangleq \overline{A} + \overline{B}_{\overline{w}}\theta(k)P^{-1}\overline{C}_{\overline{v}}\,,\\
B\left(\theta(k)\right) &\triangleq \overline{B}_d + \overline{B}_{\overline{w}}\theta(k)P^{-1}\overline{D}_{d\overline{v}}\,,\\
C\left(\theta(k)\right) &\triangleq \overline{C}_{\overline{z}} + \overline{D}_{\overline{w}\overline{z}}\theta(k)P^{-1}\overline{C}_{\overline{v}}\,,\\
D\left(\theta(k)\right) &\triangleq \overline{D}_{d\overline{z}} + \overline{D}_{\overline{w}\overline{z}}\theta(k)P^{-1}\overline{D}_{d\overline{v}}\,.
\end{aligned}$$

The model in (10) corresponds to a Markov Jump Linear System (MJLS; see [14]). We take $\theta(k)$ as the state of the associated Markov chain at time instant k which, given Assumption 3, takes values in the finite set $\mathcal{M} \triangleq \{M_1, ..., M_4\}$, where $M_1 = \operatorname{diag}\{I_{n_1}, I_{n_2}\}$, $M_2 = \operatorname{diag}\{I_{n_1}, 0_{n_2}\}$, $M_3 = \operatorname{diag}\{0_{n_1}, I_{n_2}\}$ and $M_4 = \operatorname{diag}\{0_{n_1}, 0_{n_2}\}$. By using Assumption 3 it follows that the associated transition probabilities satisfy

$$\mathcal{P}\left\{\theta(k+1) = M_j | \theta(k) = M_i\right\} = \alpha_j, \tag{11}$$

where $\alpha_1 \triangleq p_1p_2$, $\alpha_2 \triangleq p_1(1-p_2)$, $\alpha_3 \triangleq (1-p_1)p_2$ and $\alpha_4 \triangleq (1-p_1)(1-p_2)$. We also define $\mu_i \triangleq \mathcal{P}\left\{\theta(0) = M_i\right\}$, $i \in \{1, ..., 4\}$.

To simplify the notation we define, for any $M_i \in \mathcal{M}$, $A_i \triangleq A(M_i)$, $B_i \triangleq B(M_i)$, $C_i \triangleq C(M_i)$, $D_i \triangleq D(M_i)$ and, for any two families of matrices $X = \{X_1, ..., X_4\}$, $Y = \{Y_1, ..., Y_4\}$, $X_i \in \mathbb{R}^{m_x \times n_x}$, $Y_i \in \mathbb{R}^{m_y \times n_y}$

$$\eth(XTY) \triangleq \sum_{j=1}^{4} \alpha_j X_j T Y_j\,, \tag{12}$$

for any constant matrix $T \in \mathbb{R}^{n_x \times m_y}$.

A. Instantaneous second order moments

With the definitions given above, we can state the first result of this paper:

***Theorem** 1:* Consider the LTI system in Fig. 2(a) where the channel is a two-block SNR constrained AWN channel, and the MJLS in Fig. 2(b) where the link between $\bar{v}$ and $\bar{w}$ is given by (7) with P^{-1} and θ as in (8) and (9). If Assumptions 2 and 3 hold, and the distribution of $\theta(0)$ is such that $\mu_i = \alpha_i$, then, $\forall k \in \mathbb{N}_0$: [1]

1) $\mu_{\bar{x}}^L(k) = \mu_{\bar{x}}^M(k) = \mu_{\bar{x}}(k) \triangleq A_p^k \overline{\mu}_0$.
2) If, in addition, we have in Fig. 2(a) that

$$P_{q_1}(k) = p_1^{-1}(1-p_1) P_{\bar{v}_1}^L(k), \tag{13a}$$
$$P_{q_2}(k) = p_2^{-1}(1-p_2) P_{\bar{v}_2}^L(k), \tag{13b}$$

then, $\forall \tau \in \mathbb{N}_0$, $R_{\bar{x}}^L(k+\tau,k) = R_{\bar{x}}^M(k+\tau,k) = R_{\bar{x}}(k+\tau,k) \triangleq A_p^\tau P_{\bar{x}}(k)$, where $P_{\bar{x}}$ satisfies the recursion

$$\begin{aligned} P_{\bar{x}}(k+1) = {} & A_p P_{\bar{x}}(k) A_p^T + B_p P_d B_p^T \\ & + \sum_{\ell=1}^{2} p_\ell^{-1}(1-p_\ell) \overline{B}_{\bar{w}_\ell} \left(\overline{C}_{\bar{v}_\ell} P_{\bar{x}}(k) \overline{C}_{\bar{v}_\ell}^T + \overline{D}_{d\bar{v}_\ell} P_d \overline{D}_{d\bar{v}_\ell}^T \right) \overline{B}_{\bar{w}_\ell}^T , \end{aligned} \tag{14}$$

with $P_{\bar{x}}(0) = \overline{P}_0$.

Proof:

1) Immediate from (5), (10), the fact that $\mu_i = \alpha_i$, and Assumptions 2 and 3.
2) Suppose, without loss of generality, that $\overline{\mu}_0 = 0$. Then, (10), Assumptions 2 and 3, and the fact that $\mu_i = \alpha_i$ yield, $\forall k \in \mathbb{N}_0$,

$$P_{\bar{x}}^M(k+1) = \eth\left(A P_{\bar{x}}^M(k) A^T\right) + \eth\left(B P_d B^T\right). \tag{15}$$

multline Using Fact 1 from the Appendix in (15) we obtain

$$\begin{aligned} P_{\bar{x}}^M(k+1) = {} & A_p P_{\bar{x}}^M(k) A_p^T + B_p P_d B_p^T \\ & + \sum_{\ell=1}^{2} p_\ell^{-1}(1-p_\ell) \overline{B}_{\bar{w}_\ell} \left(\overline{C}_{\bar{v}_\ell} P_{\bar{x}}^M(k) \overline{C}_{\bar{v}_\ell}^T + \overline{D}_{d\bar{v}_\ell} P_d \overline{D}_{d\bar{v}_\ell}^T \right) \overline{B}_{\bar{w}_\ell}^T . \end{aligned} \tag{16}$$

Similarly, use of Assumption 2 and Fact 1 in (5) yields

$$P_{\bar{x}}^L(k+1) = A_p P_{\bar{x}}^L(k) A_p^T + B_p P_d B_p^T + \overline{B}_{\bar{w}_1} P_{q_1}(k) \overline{B}_{\bar{w}_1}^T + \overline{B}_{\bar{w}_2} P_{q_2}(k) \overline{B}_{\bar{w}_2}^T \tag{17}$$

We note that (17) is equivalent to

$$\begin{aligned} P_{\bar{x}}^L(k+1) = {} & A_p P_{\bar{x}}^L(k) A_p^T + B_p P_d B_p^T \\ & + \overline{B}_{\bar{w}} \operatorname{diag}\left\{ P_{q_1}(k) - p_1^{-1}(1-p_1) P_{\bar{v}_1}^L(k), P_{q_2}(k) - p_2^{-1}(1-p_2) P_{\bar{v}_2}^L(k) \right\} \overline{B}_{\bar{w}}^T \\ & + \sum_{\ell=1}^{2} p_\ell^{-1}(1-p_\ell) \overline{B}_{\bar{w}_\ell} \left(\overline{C}_{\bar{v}_\ell} P_{\bar{x}}^L(k) \overline{C}_{\bar{v}_\ell}^T + \overline{D}_{d\bar{v}_\ell} P_d \overline{D}_{d\bar{v}_\ell}^T \right) \overline{B}_{\bar{w}_\ell}^T , \end{aligned} \tag{18}$$

where we used the fact that (see (4) and (6))

$$P_{\bar{v}_1}^L(k) = \overline{C}_{\bar{v}_1} P_{\bar{x}}^L(k) \overline{C}_{\bar{v}_1}^T + \overline{D}_{d\bar{v}_1} P_d \overline{D}_{d\bar{v}_1}^T, \tag{19a}$$
$$P_{\bar{v}_2}^L(k) = \overline{C}_{\bar{v}_2} P_{\bar{x}}^L(k) \overline{C}_{\bar{v}_2}^T + \overline{D}_{d\bar{v}_2} P_d \overline{D}_{d\bar{v}_2}^T. \tag{19b}$$

From (18) and (16) we conclude that, if (13) hold, then, $\forall k \in \mathbb{N}_0$, $P_{\bar{x}}^L(k) = P_{\bar{x}}^M(k) = P_{\bar{x}}(k)$, where $P_{\bar{x}}$ satisfies (14). To complete the proof we note that (5) and our assumptions yield

$$R_{\bar{x}}^L(k+\tau,k) = A_p^\tau P_{\bar{x}}^L(k), \quad \tau \in \mathbb{N}_0. \tag{20}$$

On the other hand, for the MJLS, we have from (10) and Assumptions 2 and 3 that

$$R_{\bar{x}}^M(k+\tau,k) = A_p^\tau P_{\bar{x}}^M(k). \tag{21}$$

Hence, since (20) and (21) hold and $P_{\bar{x}}^L(k) = P_{\bar{x}}^M(k) = P_{\bar{x}}(k)$, the proof is completed. ■

The following immediate consequence of Theorem 1 is also relevant:

***Corollary** 1:* Consider the setup and assumptions of Theorem 1. Then, $\forall k, \tau \in \mathbb{N}_0$, $\mu_{\bar{z}}^L(k) = \mu_{\bar{z}}^M(k)$ and $R_{\bar{z}}^L(k+\tau,k) = R_{\bar{z}}^M(k+\tau,k)$. ■

[1] We use the superscripts L (resp. M) to refer to quantities related with the LTI system of Fig. 2(a) (resp. related to the MJLS of Fig. 2(b)).

Since Theorem 1 and Corollary 1 hold, we conclude that, provided (13) and some mild additional assumptions are satisfied, there exists an instantaneous second order moment equivalence between the LTI system of Fig. 2(a) and the MLJS of Fig. 2(b).

***Remark** 1:* The equivalence revealed by Theorem 1 and Corollary 1 is valid for any proper LTI system N satisfying Assumption 2(d). Accordingly, the equivalence applies, for instance, to the feedback scheme of Fig. 1, and also when in that scheme K is replaced by an LTI dynamic controller whose input is any measurable plant output. $\square$

So far, we have established an instantaneous second order moment equivalence between a class of MJLS and LTI systems subject to SNR constraints. However, so far our results do not hold for the stationary case. This issue is addressed in the next section.

B. Stability and stationary second order moments

We focus on the following notion of stability :

***Definition** 2:* Consider the MJLS in (10) with θ as in (9), initial state $\overline{x}(0) = \overline{x}_0$, $\theta(0) = \theta_0 \in \mathcal{M}$, and suppose that Assumptions 2 and 3 hold. The system in (10) is stable in the mean square sense (MSS) if and only if there exist finite $\beta \in \mathbb{R}^{\overline{n}}$ and finite $S \in \mathbb{R}^{\overline{n}\times\overline{n}}$, $S \geq 0$, such that

$$\lim_{k\to\infty} \mathcal{E}\left\{\overline{x}(k)\right\} = \beta, \quad \lim_{k\to\infty} \mathcal{E}\left\{\overline{x}(k)\overline{x}(k)^T\right\} = S,$$

regardless of $(\overline{x}_0, \theta_0)$. $\square$

It has been shown in [22] that MSS is equivalent to other common notions of stability for MJLS [23, 16], which are collectively referred to as second-moment stability. We also note that, for LTI systems, MSS is equivalent to internal stability [24] (in the standard sense defined in, e.g., [25]).

The next theorem shows how the internal stability of the LTI system of Fig. 2(a) is related to the MSS of the MJLS of Fig. 2(b):

***Theorem** 2:* Consider the LTI system in Fig. 2(a) where the channel is a two-block SNR constrained AWN channel, and the MJLS in Fig. 2(b) where the link between $\bar{v}$ and $\bar{w}$ is given by (7) with P^{-1} and θ as in (8) and (9). If Assumptions 2 and 3 hold, then:

1) If the LTI system is internally stable, $P_{q_i} = p_i^{-1}(1-p_i)P^L_{\overline{v}_i}$, $i \in \{1,2\}$, and $T_{d\overline{v}}P_dT^H_{d\overline{v}} > 0$, where $T_{d\overline{v}}$ denotes the transfer function from v to d in the system of Fig. 2(a), then the MJLS is MSS.
2) If the MJLS is MSS, then the LTI system is internally stable and there exists a choice for P_{q_i}, namely $P_{q_i} = p_i^{-1}(1-p_i)P^M_{\overline{v}_i}$, such that $P^L_{\overline{x}} = P^M_{\overline{x}}$.

Proof:

1) If the LTI system is internally stable, then $P^L_{\overline{v}}$ exists and, since $T_{d\overline{v}}P_dT^H_{d\overline{v}} > 0$, satisfies

$$P^L_{\overline{v}} > \frac{1}{2\pi}\int_{-\pi}^{\pi} T_{q\overline{v}}P_qT^H_{q\overline{v}}d\omega. \tag{22}$$

Consider a spectral factorization for the covariance matrix of q, $P_q = \Omega_q\Omega_q^H$, and note that the transfer function from q to $\overline{v}$ in Fig. 2(a), i.e., $T_{q\overline{v}}$, has a realization given by $\left(A_p, \overline{B}_{\overline{w}}, \overline{C}_{\overline{v}}, 0\right)$. Thus, Parseval's Theorem allows one to conclude from (22) that

$$P^L_{\overline{v}} - \overline{C}_{\overline{v}}\left(\sum_{k=1}^{\infty} A_p^{k-1}\overline{B}_{\overline{w}}\Omega_q\left(A_p^{k-1}\overline{B}_{\overline{w}}\Omega_q\right)^H\right)\overline{C}^T_{\overline{v}} > 0. \tag{23}$$

Since $P_{q_i} = p_i^{-1}(1-p_i)P^L_{\overline{v}_i}$, we have that

$$P_q = U\ \mathrm{diag}\left\{P^L_{\overline{v}_1}, P^L_{\overline{v}_2}\right\} = U\left(\eta_1P^L_{\overline{v}}\eta_1 + \eta_2P^L_{\overline{v}}\eta_2\right).$$

where

$$U \triangleq \mathrm{diag}\left\{p_1^{-1}(1-p_1)I_{n_1}, p_2^{-1}(1-p_2)I_{n_2}\right\}\ ,$$
$$\eta_1 \triangleq \mathrm{diag}\left\{I_{n_1}, 0_{n_2}\right\}\ , \quad \eta_2 \triangleq \mathrm{diag}\left\{0_{n_1}, I_{n_2}\right\}\ .$$

Therefore, (23) can be written as

$$P^L_{\overline{v}} - \overline{C}_{\overline{v}}L\overline{C}^T_{\overline{v}} > 0 \tag{24}$$

where $L \geq 0$ is the unique solution of

$$A_pLA_p^T - L + \overline{B}_{\overline{w}}U\left(\eta_1P^L_{\overline{v}}\eta_1 + \eta_2P^L_{\overline{v}}\eta_2\right)\overline{B}^T_{\overline{w}} = 0. \tag{25}$$

(Since A_p is Hurwitz, the series in (23) actually converges to $L \geq 0$ satisfying (25).)

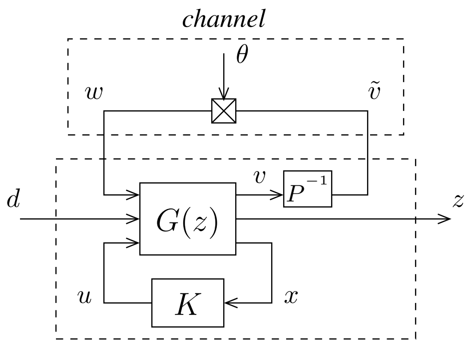

Fig. 3. Multivariable system closed over an i.i.d. analog erasure channel.

Given the above, the result follows upon proceeding as in the proof of Theorem 3.2 in [26], and using Fact 1 in the Appendix.

2) If the MJLS is MSS then Part 1 of Theorem 1 implies that A_p is Hurwitz, and thus the LTI system is internally stable (see (5)). In addition, the MSS of the MJLS also implies that $P^M_{\overline{x}}$ exist (and is unique). Hence, $P^M_{\overline{v}}$ exists and the choice proposed for P_{q_i} is a valid one.
Now, since the LTI system is stable and $(\overline{x}_0, d)$ and q satisfies Definition 1 and Assumptions 2, we have that $P^L_{\overline{x}}$ exists, and so does $P^L_{\overline{v}}$. Given the proposed choice for P_{q_i}, and since (18) holds, we conclude that $P^L_{\overline{x}} = P^M_{\overline{x}}$. ■

Theorem 2 establishes a relationship between the stability of the LTI system of Fig. 2(a) and that of the MJLS of Fig. 2(b). Note that, in Part 2), $P^L_{\overline{x}} = P^M_{\overline{x}}$ implies that the stationary variances of corresponding signals in Fig. 2(a) and Fig. 2(b) are equal. In particular this condition ensures that the stationary variance $\sigma^2_{\overline{z}}$ of $\overline{z}$ is equal in both the considered LTI and MJLS setups.

***Corollary** 2:* Consider the setup and assumptions of Theorem 2 and assume, in addition, that the MJLS of Fig. 2(b) is MSS and that the LTI system of Fig. 2(a) is internally stable. If the Markov chain governing θ in Fig. 2(b) has an arbitrary initial distribution, and the variance matrix of P_{q_i} in the LTI system of Fig. 2(a) satisfies $P_{q_i} = p_i^{-1}(1-p_i)P^L_{\overline{v}_i}$, $i \in \{1,2\}$, then $P^L_{\overline{x}} = P^M_{\overline{x}}$.

Proof: The result follows upon exploiting Definition 2, (16),[2] (17), and using a reasoning similar to that used to prove Part 2 of Theorem 2. ■

We see from Corollary 2 that, if we relax the conditions of Theorem 1 so as to allow for arbitrary initial distributions of θ, and impose (13) in steady state only, then a *stationary* second order moment equivalence arises between the LTI system of Fig. 2(a) and the MJLS system of Fig. 2(b). The instantaneous equivalence revealed by Theorem 1 and Corollary 1 requires stronger conditions.

With the results summarized above, we are now in a position to present the second contribution of this paper.

V. Upper bounds on the best achievable performance subject to SNR constraints

In this section we use the result of Section IV to provide upper bounds on the solution of Problem 1, i.e., upper bounds on $[\sigma^2_z]_{\Gamma_1,\Gamma_2}$. To do so, we introduce the following assumption:

***Assumption** 4:* In Problem 1, the stationary SNR constraints are active at the optimum. □

Assumption 4 is reasonable. It holds, for example, when $n_1 = n_2 = 1$ and both the closed loop transfer function from the disturbance d to v_1, and from d to v_2 (see Definition 1) are non-zero at the optimum. The latter conditions are tantamount to assuming that the optimal state feedback gain is such that the channels associated with both v_1 and v_2 do actually transmit information about the disturbance across them.

To proceed, we start by noting that Assumption 1 guarantees that the system of Fig. 1, when rewritten as in Fig. 2(a), satisfies Assumption 2. Thus, Theorems 1 and 2, and Corollaries 1 and 2 apply to the system of interest in Problem 1. Accordingly, we consider the MJLS of Fig. 3, where the two-block SNR constrained AWN channel of Fig. 1 has been replaced by the auxiliary channel described by (7)-(9).

Given Assumption 1(e) we have that a state space description of the (open loop) system of Fig. 3, when no feedback from x to u is present, is given by

$$x(k+1) = \mathcal{A}\left(\theta(k)\right)x(k) + \mathcal{B}\left(\theta(k)\right)u(k) + \mathcal{H}\left(\theta(k)\right)d(k), \tag{26a}$$

$$z(k) = \mathcal{C}\left(\theta(k)\right)x(k) + \mathcal{D}\left(\theta(k)\right)u(k), \tag{26b}$$

[2]Even if $\mu_i \neq \alpha_i$ (16) holds for $k \geq 1$ (θ is an i.i.d. sequence).

where all matrices depend on the state space description of G in (3), and on the matrix gain P^{-1}. Denote by $\mathcal{G}_{\rm CL}$ the MJLS that arises when (26) is considered together with the static state feedback control law $u = Kx$ (i.e., $\mathcal{G}_{\rm CL}$ corresponds to the MJLS of Fig. 3). We define the 2-norm of $\mathcal{G}_{\rm CL}$ as follows (see [14]):

***Definition** 3:* If $\mathcal{G}_{\rm CL}$ is MSS, then its 2-norm is defined by

$$\|\mathcal{G}_{\rm CL}\|_2^2 \triangleq \sum_{s=1}^{p} \left|\left|z^{(s)}\right|\right|^2 \tag{27}$$

where $z^{(s)}$ represent the output sequence $\{z(k); k \in \mathbb{N}_0\}$ when

- $x(0) = 0$ and $d(k) = e_s\delta(k)$, where $e_s \in \mathbb{R}^p$ is the s^{th} column of the $p \times p$ identity matrix, $\delta(k)$ is a Kronecker delta, and
- $\theta(0) = M_i \in \mathcal{M}$, with probability $\mu_i > 0$. □

With the previous definitions we can state the following consequence of the results of Section IV:

***Corollary** 3:* Consider Problem 1 and the MJLS $\mathcal{G}_{\rm CL}$ defined above. If θ is as in (9) and Assumptions 3 and 4 hold, then

$$\left[\sigma_z^2\right]_{\Gamma_1,\Gamma_2} = \inf_{K \in \mathcal{S}^M} \|\mathcal{G}_{\rm CL}\|_2^2 \,, \tag{28}$$

where

$$\mathcal{S}^M \triangleq \left\{K \in \mathbb{R}^{m\times n} : \mathcal{G}_{\rm CL} \text{ is MSS}\right\}.$$

Proof: From the results in Section 4.4.3 of [14], we have that $\|\mathcal{G}_{\rm CL}\|_2^2 = \lim_{k\to\infty} \mathcal{E}\left\{z(k)^T z(k)\right\} = \sigma_z^2$. The results thus follows upon exploiting Theorem 2, Corollary 2, and Assumption 4. ■

Corollary 3 shows that, under mild assumptions, solving Problem 1 is equivalent to solving an optimal control problem for the MJLS of Fig. 3. Such problems have received much attention in the literature (see, e.g., [14, 16–19] and the references therein). However, most results in MJLS theory assume that the controller is mode-dependent, i.e., that it has available measurements of the state of the associated Markov chain [14]. The latter assumption is not compatible with the situation of interest in this paper. Indeed, in Problem 1 we are interested in static, time-invariant, state feedback gains K, which implies that K in Fig. 3 cannot depend on θ. Although the literature on optimal mode-independent controller design is limited, we can use the results in [19] to state the following:

***Theorem** 3:* Consider Problem 1, suppose that Assumption 4 holds, and use the notation introduced in (26) to define the following convex optimization problem in the variables $W_1, ..., W_4$, $R_1, ..., R_4$, Q and F:

$$\text{Find:} \quad \rho \triangleq \inf \sum_{i=1}^{4} \text{trace}\left\{W_i\right\} \tag{29}$$

$$\text{s.t.} \quad \begin{bmatrix} W_i & \mathcal{C}_i Q + \mathcal{D}_i F \\ Q^T\mathcal{C}_i^T + F^T\mathcal{D}_i^T & Q + Q^T - \alpha_i \sum_{j=1}^4 R_j \end{bmatrix} > 0\,, \tag{30}$$

$$\begin{bmatrix} R_i - \frac{1}{4}\mathcal{H}_i\mathcal{H}_i^T & \mathcal{A}_i Q + \mathcal{B}_i F \\ Q^T\mathcal{A}_i^T + F^T\mathcal{B}_i^T & Q + Q^T - \alpha_i \sum_{j=1}^4 R_j \end{bmatrix} > 0\,, \tag{31}$$

where $i \in \{1, ..., 4\}$, α_i is as in (11), and $\mathcal{A}_i \triangleq \mathcal{A}(\theta(k))$, $\mathcal{B}_i \triangleq \mathcal{B}(\theta(k))$, $\mathcal{H}_i \triangleq \mathcal{H}(\theta(k))$, $\mathcal{C}_i \triangleq \mathcal{C}(\theta(k))$, $\mathcal{D}_i \triangleq \mathcal{D}(\theta(k))$, when $\theta(k) = M_i \in \mathcal{M}$ (recall the definitions introduced before (11)).

If the problem in (29)-(31) is feasible, then

$$\left[\sigma_z^2\right]_{\Gamma_1,\Gamma_2} \le \rho. \tag{32}$$

Moreover, if F_o and Q_o are the optimal values of F and Q, then the choice $K = K_o \triangleq F_o Q_o^{-1}$ and $P_{q_i} = \Gamma_i^{-1} P_{v_i}^M$, $i \in \{1, 2\}$, where $P_{v_i}^M$ is the stationary variance of v in the MJLS of Fig. 3 when $K = K_o$, guarantees the internal stability of the LTI system of Fig. 1, and also that the stationary variance of z in that system satisfies $\sigma_z^2 \le \rho$.

Proof: If we assume, without loss of generality, that the distribution of $\theta(0)$ satisfies $\mu_i = \frac{1}{4}$, then the definition of 2-norm in [19] reduces to the 2-norm in Definition 3. Thus, the results of [19] apply to the optimization problem on the right hand side of (28), and (32) follows from Theorem 6 in [19] (see also the remark before Section 4 in [19]). The results in [19] also guarantee that $K_o \in \mathcal{S}^{\mathcal{M}}$. Hence, our remaining claims follow from Theorem 2, Corollary 2, and from the fact that the MSS of the MJLS of Fig. 3 implies that $P_{v_i}^M$ exists. ■

Provided the convex optimization problem in (29)-(31) is feasible, Theorem 3 gives an upper bound on the best achievable performance in the networked system of Fig. 1, when stationary SNR constraints are imposed on the communication channels. As such, Theorem 3 gives an upper bound on the solution to Problem 1.

The optimization problem in (29)-(31) involves LMIs and, as such, it can be addressed using standard algorithms [20, 27]. It is also worth noting that the feasibility of the LMIs in (30)-(31) is only sufficient for the existence of feedback gains in $\mathcal{S}^{\mathcal{L}}$. Therefore, the optimization problem in (29)-(31) may be unfeasible and, nevertheless, Problem 1 may admit a solution.

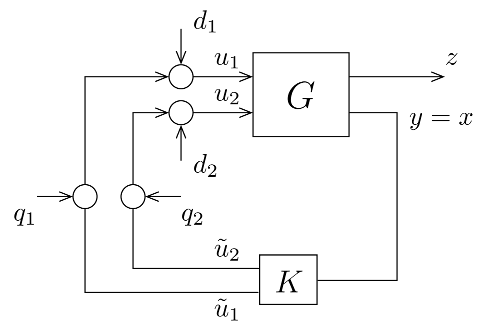

Fig. 4. Networked systems considered in Section VI.

***Remark** 2:* The problem of optimal mode-independent dynamic output feedback controller design for MJLS, to the best of the authors' knowledge, is still an open problem [16]. If a solution to this problem did exist, then the extension of the results in this section to the dynamic output feedback case would be immediate. □

VI. Simulation study

This section presents an example that illustrates the use of the ideas in this paper to design static state feedback controllers, when the channels used to transmit the control signals are subject to stationary SNR constraints.

Consider the system of Fig. 4. For this system, we want to design a static controller K which feds back the state x of the multivariate plant G described by

$$G = \begin{bmatrix} \frac{1}{z-0.8} & \frac{1}{z} \\ \frac{2}{z-0.2} & \frac{1}{z-1.5} \end{bmatrix} . \tag{33}$$

The disturbance $d \triangleq \begin{bmatrix} d_1 & d_2 \end{bmatrix}^T$ is assumed to be white noise with covariance matrix $P_d = I_2$, and the control signals $\tilde{u}_1$ and $\tilde{u}_2$ are to be transmitted through two channels with the maximum admissible SNRs

$$\Gamma_1 = 1 \quad \text{and} \quad \Gamma_2 = 10\,, \tag{34}$$

respectively. Using the results of Section IV, we can alternatively focus on the system of Fig. 5, where the successful transmission probabilities in the erasure channels are $p_1 = 0.5$ and $p_2 = 0.9091$, for the channels associated with $\tilde{u}_1$ and $\tilde{u}_2$, respectively.

Using the CVX package for Matlab [27], we solved the Problem in (29)-(31) and obtained $\rho = 28.4262$ and a feedback gain

$$K = \begin{bmatrix} 0.0694 & -0.1793 & 0.0136 & -0.0734 \\ -1.2732 & 0.0578 & -0.0344 & 0.0395 \end{bmatrix} . \tag{35}$$

For this value of K, we find the variances of the noises q_i in Fig. 4 by calculating the stationary solution of (14) and using $P_{q_i} = p_i^{-1}(1-p_i)P_{v_i}^M$:

$$P_{q_1} = 0.0777 \quad \text{and} \quad P_{q_2} = 0.2068\,. \tag{36}$$

We simulated the system of Fig. 4 with the data in (35) and (36)[3], and obtained the following *measured* SNRs for each channel:

$$\frac{\sigma_{v_1}^2}{\sigma_{q_1}^2} = 0.998 \quad \text{and} \quad \frac{\sigma_{v_2}^2}{\sigma_{q_2}^2} = 9.9692\,. \tag{37}$$

The *measured* value for the stationary variance of z is found to be equal to $\sigma_z^2 = 10.4148 \leq \rho = 28.4262$.

We can see that, as expected, the measured SNRs in (37) are essentially equal to the maximum allowable SNRs in (34). We also see that since the simulated value of the stationary variance of z is upper bounded by ρ, (32) is also necessarily satisfied.

VII. Conclusions

This paper has studied two-channel networked control architectures for MIMO LTI plants, where the channels are subject to SNR constraints. As a first step in our study, we showed that the original SNR constrained optimal control problem is equivalent to an optimal control problem involving MJLSs. This enabled us to use well-known MJLS theory facts to provide upper bounds on the best achievable performance when static state feedback controllers are used in the original SNR constrained situation.

Future work should explore the use of output-feedback controllers, and/or extensions to the general n-channel case.

[3] All results are averages over 100 simulations, each 10^4 samples long.

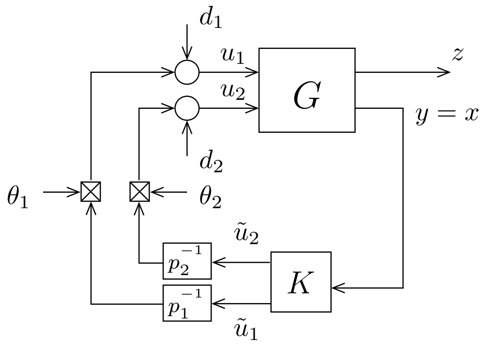

Fig. 5. Equivalent rewriting of the feedback system of Fig. 4.

VIII. ACKNOWLEDGMENTS

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APPENDIX

The following is a consequence of a straightforward but lengthy calculation:

Fact 1: Consider matrices X^0, X^1, $X^2 \in \mathbb{R}^{n_x \times m_x}$, Y^0, Y^1, $Y^2 \in \mathbb{R}^{n_y \times m_y}$, and $p_1, p_2 \in \mathbb{R}$. Define

$$X_p \triangleq X^0 + X^1 + X^2 \qquad Y_p \triangleq Y^0 + Y^1 + Y^2 .$$

and $X_1 \triangleq X^0 + p_1^{-1}X^1 + p_2^{-1}X^2$, $X_2 \triangleq X^0 + p_1^{-1}X^1$, $X_3 \triangleq X^0 + p_2^{-1}X^2$, $X_4 \triangleq X^0$, $Y_1 \triangleq Y^0 + p_1^{-1}Y^1 + p_2^{-1}Y^2$, $Y_2 \triangleq Y^0 + p_1^{-1}Y^1$, $Y_3 \triangleq Y^0 + p_2^{-1}Y^2$, $Y_4 \triangleq Y^0$. If $\alpha_1 = p_1 p_2$, $\alpha_2 = p_1(1-p_2)$, $\alpha_3 = (1-p_1)p_2$, $\alpha_4 = (1-p_1)(1-p_2)$, and T is a matrix of appropriate dimensions, then we have that for the families of matrices $X = \{X_1, ..., X_4\}$ and $Y = \{Y_1, ..., Y_4\}$, the following holds (see (12)):

$$\eth\left(XTY^T\right) = \sum_{i=1}^{4} \alpha_i X_i T Y_i^T = X_p T Y_p^T + \sum_{i=1}^{2} p_i^{-1}(1-p_i) X^i T {Y^i}^T$$

■

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